Challenge 5, Optics: Solution

All kinds of optics *15 pt.*

Parts A, B and C are independent of each other and can therefore be solved individually.

Part A. Astronaut *3 pt.*

i. An astronaut is in his spaceship in an orbit $280\,\mathrm{km}$ from the surface of the Earth. What is the minimum distance between two points on the Earth's surface so that he can still tell them apart (under optimal conditions)? The diameter of the pupil is $0.5\,\mathrm{cm}$ and the wavelength of light is $550\,\mathrm{nm}$. **3 pt.**

The minimal resolution for a circular aperture of radius R is given $\theta_{min} = \frac{1.22\lambda}{2R}$ 1 pt.

For a small angle, at a distance D, on can distinguish a distance of $d_{min} = D\theta_{min}$. 1 pt.

So our astronaut could see two objects separated by a distance $d_{min} = \frac{1.22\lambda D}{2R}$. We get $d_{min} = 38\,\mathrm{m}$. 1 pt.

Part B. Coated glasses *5 pt.*

A beam of white light is shining perpendicularly on a lens (n=1.52), which is covered with a thin film of magnesium fluoride (n=1.38).

i. What is the minimum film thickness at which the reflected light contains no yellow-green light of wavelength $550\,\mathrm{nm}$ (in air)? **3 pt.**

We consider the interference of two light rays. One is reflected at the surface between air and magnesium fluorid and the other guess through the magnesium fluorid and gets reflected at the surface between glass and magnesium fluorid.

The phase shifts at the two reflections cancel each other, due to increasing refractive indices ($1 = n_{vacuum} < n_{coating} < n_{bulk}$) 1 pt.

We have therefore an optical path difference of $\Delta s = 2ln$, where l is the thickness of the coating and n is its refractive index. 0.5 pt.

The condition for a destructive interference is $\Delta s = \lambda(k + 0.5)$, where k is an integer. 0.5 pt.

The minimal thickness is therefore $l_{min} = \frac{\lambda}{4n} = 99.6\,\mathrm{nm}$. 1 pt.

ii. For which minimum layer thickness (different from zero) does constructive interference result for the reflected light? **2 pt.**

The condition for a constructive interference is $\Delta s = k\lambda$ 1 pt.

The minimal thickness is therefore $l_{min} = \frac{\lambda}{2n} = 199\,\text{nm}$. 1 pt.

Part C. Newton's rings *7 pt.*

In 1717, Sir Isaac Newton studied an interesting phenomenon: If you approach a spherical surface to a reflecting plane surface, you observe a series of concentric rings (see figure) when you look through the glass from above. In our case, the light source is monochromatic and has a wavelength λ.

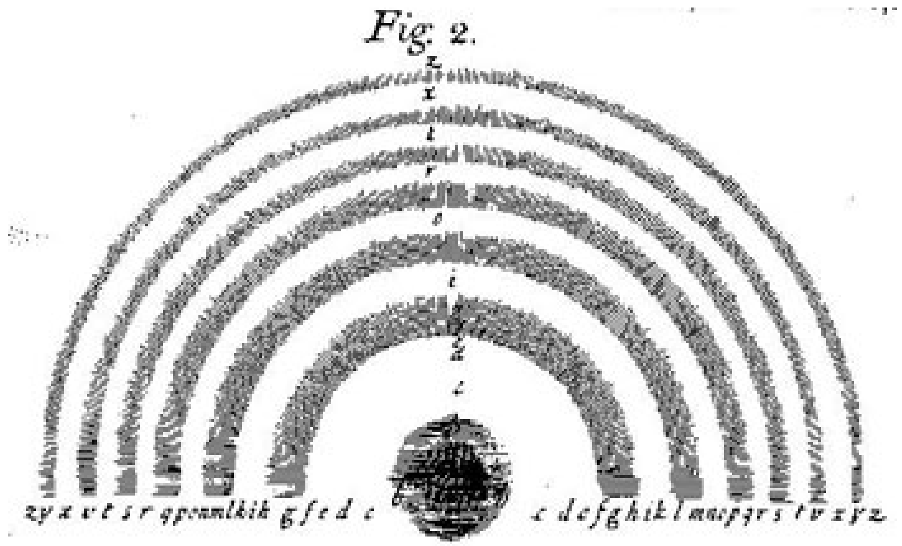

Figure 1:

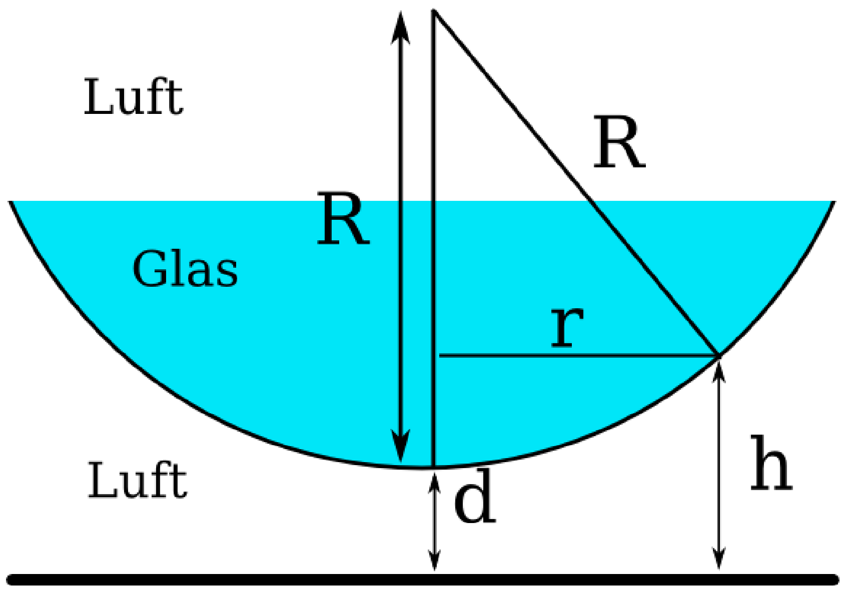

Figure 2:

i. Explain why you see rings, what the conditions are for bright rings and what the conditions are for dark rings. **4 pt.**

There is an interference between the light which is reflected at the bottom surface of the curved lens and the one which has an additional travel of two times h, the height between the curved lens and the flat surface. 2 pt.

The light going to the reflective surface picks up an additional half wavelength phase shift. 1 pt.

Therefore the condition for constructive interference reads

$$\frac{2h}{\lambda} = n + \frac{1}{2} = n + \frac{1}{2}$$

where n is an integer. For constructive interference we see a bright ring. 0.5 pt.

Therefore the condition for constructive interference reads

$$\frac{2h}{\lambda} = n' + \frac{1}{2} + \frac{1}{2} = n$$

where n' and n are integers. For destructive interference we see a dark ring. 0.5 pt.

Having been delighted by the beauty of the rings that bear his name, Sir Isaac Newton might have wondered at what distance d the lens is from the plane surface.

ii. Express d as a function of the radius R of the curved glass surface, the radius r_n of the n^{th} dark ring and the wavelength λ. **3 pt.**

We have

$$h_n = d + (R - \sqrt{R^2 - r_n^2})$$

1 pt.

With the assumption that we only look at rings close to the center $r_n \ll R$ we get

$$h_n = d + R(1 - \sqrt{1 - \frac{s^2}{R^2}}) \approx d + \frac{s^2}{R^2}$$

1 pt.

With the condition for destructive interference we get

$$d = \frac{\lambda}{2}n - \frac{r_n^2}{2R}$$

1 pt.