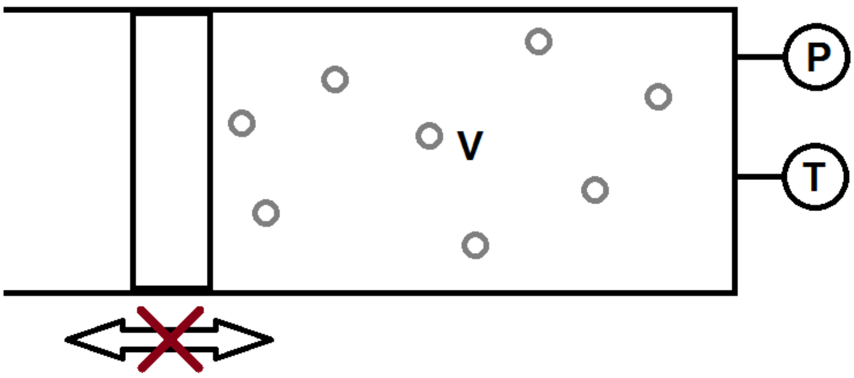
Challenge 2, Thermodynamics: Solution

Under pressure *16 pt.*



A cylinder with initial volume $V = 2\,\mathrm{L}$ is filled with nitrogen. The cylinder is hermetically sealed by a piston. In the first part of the task, the piston is blocked, the gas is heated to different temperatures and the pressure is measured for each temperature. The results of the measurements are shown in the following table.

#	T(°C)	P(Pa)
1	10	168100
2	20	174000
3	50	191800
4	100	221500
5	150	251200
6	250	310600

Part A. Determination of the mass *5 pt.*

i. Draw a graph of the pressure as a function of temperature in the cylinder. **2 pt.**

0.25 points for graph label on each axis 0.5 pt.

0.25 for correct units on each axis 0.5 pt.

correct ploting of data 1 pt.

Typically you take off : -(0.25 pt) if the data do not span over the whole axis (ie if they plot from 0 to 350000 for the pressure, from 150000 is sufficient and allows better visualization of the data) -(0.25 pt) if the graph is super small on the sheet of paper

ii. What is the mass of the nitrogen in the cylinder? **3 pt.**

The ideal gas law

$$PV = nRT$$

0.5 pt.

We substitue $n = \frac{m}{M}$ and divide by V

$$P = \frac{mR}{MV}T$$

0.5 pt.

From the slope one can determine the mass

$$m = slope \cdot \frac{MV}{R}$$

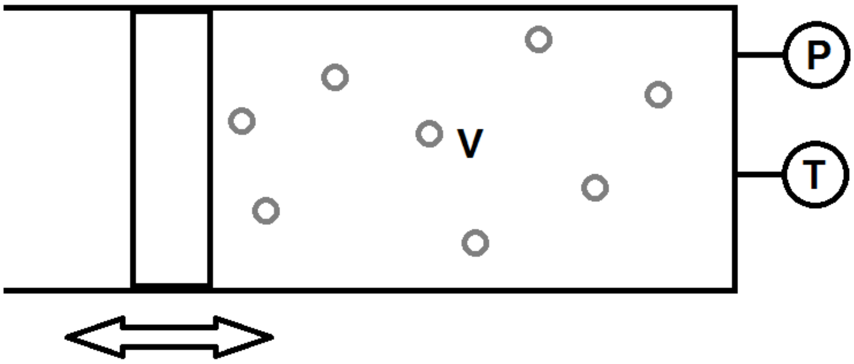
1 pt.

Get the numerical value for the slope from the graph. 0.5 pt.

For finding the numerical value of the mass (4 g). The results in the following sections will be calculated with this value. 0.5 pt.

Part B. Equilibrium *2 pt.*

Now the piston can move parallel to the cylinder axis, but it will never leave the cylinder. The heat transfer between the gas and the environment can be neglected, unless otherwise stated.



i. If the temperature of the gas is 23 °C, how big is the volume of the gas in the cylinder? **2 pt.**

We have

$$V = (mR/M_m P)T = 3.48\,\mathrm{L}$$

1 pt.

where we have used that $P = P_{atm} = 1 \times 10^5\,\mathrm{Pa}$, anything close to that within 10% is acceptable. 1 pt.

Part C. We dive *9 pt.*

A diver brings the cylinder under water, $h = 7\,\mathrm{m}$ below the water surface. The temperature would still be 23 °C and the piston can still move freely.

Constants:

- **density of water:** $\rho_w = 1.0\,\mathrm{gcm^{-3}}$
- **Heat capacity of nitrogen:** $c_s = 1.04\,\mathrm{kJkg^{-1}K^{-1}}$

i. What volume does the gas occupy if the temperature remains unchanged? **2 pt.**

The pressure in this case is

$$P = P_{atm} + \rho_{water} g h$$

1 pt.

Again we have

$$V = (mR/M_m P)T$$

0.5 pt.

The numerical application gives $V = 2.07\,\mathrm{L}$ 0.5 pt.

ii. The cylinder is left under water. What amount of heat must be added to the gas so that its volume becomes 2 L? **3 pt.**

Again we have

$$P = P_{atm} + \rho_{water} g h$$

0.5 pt.

which gives

$$(V M_m P)/(mR) = Tf = 285.69\,\mathrm{K} = 12.69\,°\mathrm{C}$$

1 pt.

we find for the heat

$$Q = mc_{nitrogen}(Tf - Ti)$$

1 pt.

Resulting in a numerical value of $Q = -42.9\,\mathrm{J}$ 0.5 pt.

iii. What must be the mass of the cylinder (as a function of the temperature of the gas) so that the cylinder remains in equilibrium at the same depth? (i.e., it neither rises back to the surface nor sinks to the bottom.) The mass of the piston will be neglected. The volume of the piston and cylinder together can also be neglected compared to the volume of the gas. **4 pt.**

Archimedes :

$$F_A = \rho_{water} V_{fluid} g$$

1 pt.

Equilibrium :

$$F_A = (m_{cylinder} + m)g$$

1 pt.

Combining the two latter (one can simplify g) :

$$\rho_{water} V_{fluid} = m_{cylinder} + m$$

0.5 pt.

$$V = (mR/M_m P)T$$

0.25 pt.

$P = Patm + \rho_{water} gh$ 0.25 pt.

it yields

$$m_{cylinder}(T) = \left(\frac{\rho_{water} R}{M_m(P_{atm} + \rho_{water} gh)} T - 1 \right) m$$

1 pt.