Challenge 3, Waves and Oscillations: Solution

Pendulum in the capacitor *14 pt.*

In this task we investigate the behaviour of a pendulum in an electric field. For this purpose we consider an (ideal) plate capacitor with plate area $A = 1\,\mathrm{m}^2$ and distance $d = 20\,\mathrm{cm}$ and a small sphere with mass $m = 5\,\mathrm{g}$, which is suspended from a thread of length $l = 10\,\mathrm{cm}$.

Part A. Electric field *3 pt.*

i. Calculate the capacitance C of the capacitor. **1 pt.**

We have

$$C = \varepsilon_0 \frac{A}{d}$$

0.5 pt.

The numerical result is $C = 44.27\,\mathrm{pF}$. 0.5 pt.

ii. What is the voltage and electric field when the charge $Q = \pm 2\,\mathrm{\mu C}$ resides on the capacitor plates? **2 pt.**

We have

$$U = \frac{Q}{C}$$

0.5 pt.

and therefore

$$E = \frac{U}{d}$$

0.5 pt.

the numerical results are

$$U = 44.52\,\mathrm{kV} \text{ and } E = 226\,\mathrm{kV}\cdot\mathrm{m}^{-1}$$

(0.5 points each) 1 pt.

Part B. Oscillation *11 pt.*

If you could not solve the previous tasks, use an electric field of $E = 2.26\,\mathrm{MV}\cdot\mathrm{m}^{-1}$ for the following tasks.

i. In the middle of the condenser, whose plates are parallel to the yz -plane (i.e. perpendicular to gravity), we now place our pendulum. The sphere is charged with $q = 200\,\mathrm{nC}$. Sketch which forces act on the sphere and the resulting force. **2 pt.**

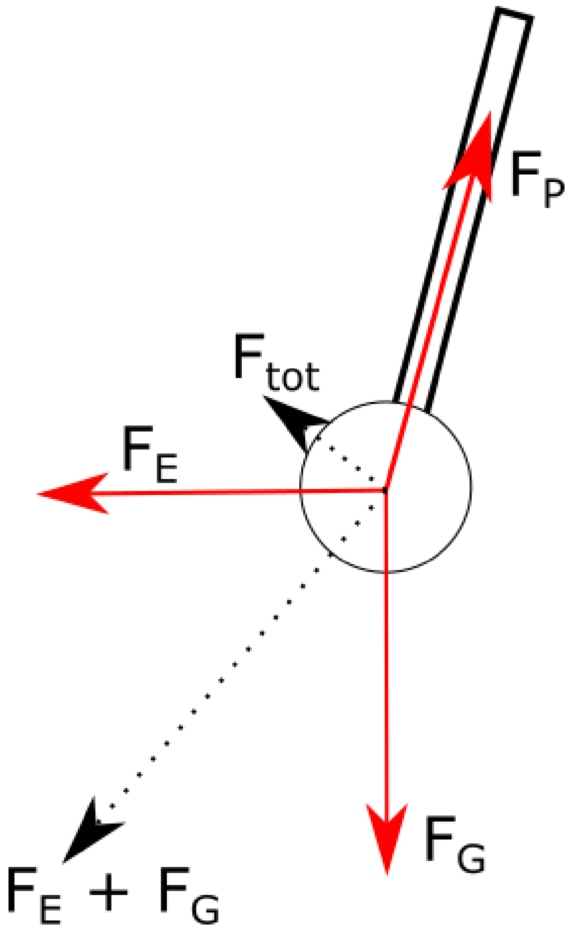

Figure 1:

For the electrical force.	0.5 pt.
For the gravititional force.	0.5 pt.
For the force in the pendulum.	0.5 pt.
For the resulting force.	0.5 pt.

ii. What angle does the pendulum assume with the vertical when at rest? **2 pt.**

In x-direction we have the electrical force $F_{el} = qE$ and in z direction the gravitational force $F_g = mg$ 0.5 pt.

Therefore we get the angle

$$\theta = \arctan\left(\frac{F_{el}}{F_g}\right)$$

1 pt.

The numerical result is $\theta = 83.8°$ (for $E = 2.26\,\mathrm{MV\cdot m^{-1}}$) or $\theta = 42.65°$ (for $E = 226\,\mathrm{kV\cdot m^{-1}}$). 0.5 pt.

iii. If the pendulum is deflected by a small angle from this rest position, it will perform an (approximately harmonic) oscillation. Calculate the frequency of this oscillation! *Note: For* $x \ll 1$ $\sin x \approx x$ *holds.* **5 pt.**

At every point we have the same force $F_{el+g} = \sqrt{F_{el}^2 + F_g^2}$ acting on the charge under an the angle θ, which we calculated above. Around the equilbrium point the force becomes

$$F_r = -F_{el+g}\sin(\phi)$$

where ϕ deflection from equilbrium. 1 pt.

We make a Taylor approximation for small ϕ

$$F_r \approx -F_{el+g}\phi$$

0.5 pt.

We get the equation of motion

$$ml\ddot{\phi} = -F_{el+g}\phi$$

and therefore

$$\ddot{\phi} = -\frac{F_{el+g}}{ml}\phi$$

1 pt.

From $\ddot{\phi} = -\omega^2\phi$ we can identify

$$\omega = \sqrt{\frac{F_{el+g}}{ml}}$$

1 pt.

With $\omega = 2\pi f$ 0.5 pt.

the frequency becomes

$$f = \frac{1}{2\pi\sqrt{ml}}\left((qE)^2 + (mg)^2\right)^{1/4} = \frac{1}{2\pi\sqrt{ml}}\left(\left(\frac{qQ}{\epsilon_0 A}\right)^2 + (mg)^2\right)^{1/4}$$

0.5 pt.

The numerical value of f is $4.8\,\mathrm{Hz}$ (for $E = 2.26\,\mathrm{MV\cdot m^{-1}}$) or $1.838\,\mathrm{Hz}$ (for $E = 226\,\mathrm{kV\cdot m^{-1}}$) 0.5 pt.

iv. How does the frequency of the oscillation change if the voltage of the capacitor is kept constant and the distance between the plates is increased by $\Delta d = 5\,\mathrm{cm}$? **2 pt.**

The charge changes by

$$Q' = Q\frac{d}{d + \Delta d}$$

1 pt.

Therefore we can use the formula

$$f = \frac{1}{2\pi\sqrt{ml}}\left(\left(\frac{qQ}{\epsilon_0 A}\right)^2 + (mg)^2\right)^{1/4}$$

again. 0.5 pt.

The numerical value is $f = 4.3\,\mathrm{Hz}$ (for $E = 2.26\,\mathrm{MV}\cdot\mathrm{m}^{-1}$) or $f = 1.757\,\mathrm{Hz}$ (for $E = 226\,\mathrm{kV}\cdot\mathrm{m}^{-1}$) 0.5 pt.