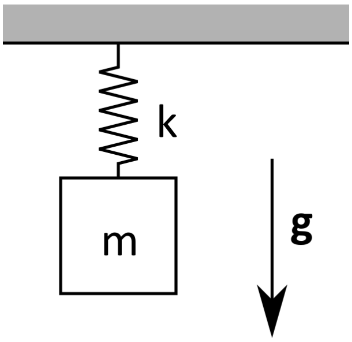
Waves and Oscillations

Warm-Up questions

Harmonic oscillations (Chapter 5.2)

i. A mass is hung from the ceiling with a spring. Find a function that describes the vertical position of the mass m as a function of time. The spring has a spring constant k and is initially unstretched.

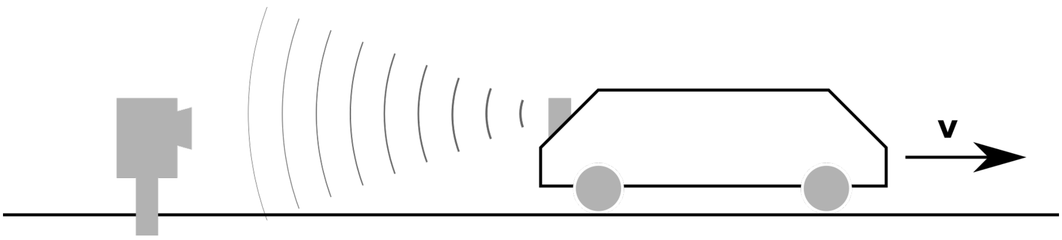


Hint: Begin by drawing a coordinate system and choosing your axes. Designate the equilibrium position of the spring with x_0. Describe the system using a differential equation and modify the equation to obtain a homogeneous differential equation (meaning that the equation only contains terms that are dependent on x or derivatives of x). Solve the equation using the approach described in the script.

Waves propagation (Chapter 6.4)

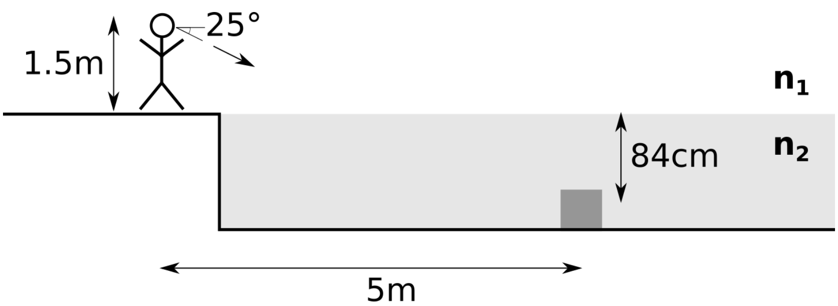
ii. In your own words, explain why the pitch of an ambulance siren increases as the ambulance approaches you?

iii. Maxime wants to measure the maximal speed of his electric train. To do this, he places a small alarm on his train that generates a tone of frequency $f_1 = 1\,\text{kHz}$. He also places a small sensor on the train tracks that can measure the frequency. While the train moves away from the sensor, the sensor measures a frequency of $f_2 = 994\,\text{Hz}$. How fast is the train moving? The speed of sound in air is $v = 340\,\text{m/s}$.



Waves propagation at interfaces (Chapter 6.5)

iv. Alice is standing at the edge of a pool and looks at the water at an angle of $\phi = 25°$ below the horizon. She has previously placed an object 84 cm below the water and 5 m away from the edge where she is standing.



a) Assuming there was no water, is Alice looking in the right direction (and at the right angle) to see the object?

b) How does your answer to part a change if we consider that there is water in the pool?

c) Assume that Alice is not looking in the right direction, should she raise her head (the angle below the horizon becomes smaller) or lower her head to see the object?

Multi-Waves phenomena (Chapter 6.6)

v. Calculate the period of the following wave:

$$y(t) = \cos(10\pi t) - \sin(15\pi t) + \sin(20\pi t + \pi/2)$$

vi. At what frequencies can a guitar string of length $L = 90\,\mathrm{cm}$ vibrate? How does your answer change if we consider a pipe of the same length that is closed on one end and open on the other?

Hint: In both cases, we are dealing with stationary waves, meaning waves that seem to oscillate in place without propagating forwards. This is due to the presence of two identical waves that are propagating in opposite directions. For the guitar string, assume that the two ends of the string are fixed and cannot move. For the pipe, assume that the pressure at the open end of the pipe is constant