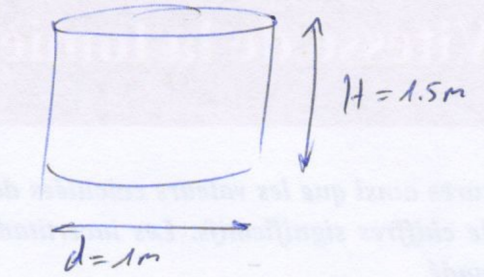
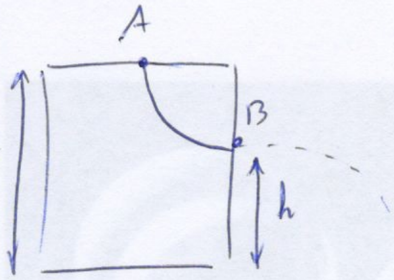
Hydrostatics



-0.5
pas math error

2 i)



Bernoulli:

$$\frac{1}{2}\rho v_A^2 + \rho g h_A + p_A = \frac{1}{2}\rho v_B^2 + \rho g h_B + p_B$$ (1)

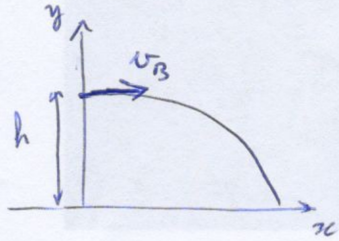
- $p_A \sim p_B \sim p_{atm}$
- $v_A \sim 0$

(0.5)

$$\Rightarrow \quad \rho g H = \frac{1}{2}\rho v_B^2 + \rho g h$$

$$\sim \quad \boxed{v_B = \sqrt{2g(H-h)}}$$ (0.5)

1 ii)



$$x(t) = x_0 + v_0 t = v_B t$$
$$y(t) = y_0 + v_0 t - \frac{1}{2}gt^2 = h - \frac{1}{2}gt^2$$ (0.5)

$$y(t_{fin}) = 0 \Rightarrow t_{fin} = \sqrt{\frac{2h}{g}}$$

(0.5) t_{fin} in x $\Rightarrow$
$$\boxed{x(t_{fin})} = v_B t_{fin} = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}} = \boxed{2\sqrt{h(H-h)}}$$

1 iii) $x(h) = 2\sqrt{h(H-h)}$

$$x_{max} \Leftrightarrow x'(h) = 0$$ (0.5)
$$\Leftrightarrow 2 \cdot \frac{1}{2}\left(h(H-h)\right)^{-1/2}(H-2h) = 0$$
$$\Leftrightarrow \boxed{h = H/2}$$ (0.5)

(this is indeed a maximum, e.g. $x'\left(\left(\frac{H}{2}\right)^-\right) > 0$
$x'\left(\left(\frac{H}{2}\right)^+\right) < 0$) not necessary