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# Physics Olympiad

## Final Round

18 - 19 March 2023

| | | |
|---:|:---:|:---|
| **Part 1** | **:** | **3 long problems** |
| Duration | : | 150 minutes |
| Total | : | 48 points ($3 \times 16$) |
| Authorized material | : | Calculator without database |
| | | Writing and drawing material |

# Good luck!


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#e0650292

## Natural constants

| | | | | |
|---|---|---|---|---|
| Caesium hyperfine frequency | $\Delta\nu_{\mathrm{Cs}}$ | 9.192 631 770 | $\times 10^{9}$ | $\mathrm{s^{-1}}$ |
| Speed of light in vacuum | $c$ | 2.997 924 58 | $\times 10^{8}$ | $\mathrm{m \cdot s^{-1}}$ |
| Planck constant | $h$ | 6.626 070 15 | $\times 10^{-34}$ | $\mathrm{kg \cdot m^{2} \cdot s^{-1}}$ |
| Elementary charge | $e$ | 1.602 176 634 | $\times 10^{-19}$ | $\mathrm{A \cdot s}$ |
| Boltzmann constant | $k_{\mathrm{B}}$ | 1.380 649 | $\times 10^{-23}$ | $\mathrm{K^{-1} \cdot kg \cdot m^{2} \cdot s^{-2}}$ |
| Avogadro constant | $N_{\mathrm{A}}$ | 6.022 140 76 | $\times 10^{23}$ | $\mathrm{mol^{-1}}$ |
| Luminous efficacy of radiation | $K_{\mathrm{cd}}$ | 6.83 | $\times 10^{2}$ | $\mathrm{cd \cdot kg^{-1} \cdot m^{-2} \cdot s^{3} \cdot sr}$ |
| Magnetic constant | $\mu_{0}$ | 1.256 637 062 12(19) | $\times 10^{-6}$ | $\mathrm{A^{-2} \cdot kg \cdot m \cdot s^{-2}}$ |
| Electric constant | $\varepsilon_{0}$ | 8.854 187 812 8(13) | $\times 10^{-12}$ | $\mathrm{A^{2} \cdot kg^{-1} \cdot m^{-3} \cdot s^{4}}$ |
| Gas constant | $R$ | 8.314 462 618... | | $\mathrm{K^{-1} \cdot kg \cdot m^{2} \cdot mol^{-1} \cdot s^{-2}}$ |
| Stefan-Boltzmann constant | $\sigma$ | 5.670 374 419... | $\times 10^{-8}$ | $\mathrm{K^{-4} \cdot kg \cdot s^{-3}}$ |
| Gravitational constant | $G$ | 6.674 30(15) | $\times 10^{-11}$ | $\mathrm{kg^{-1} \cdot m^{3} \cdot s^{-2}}$ |
| Electron mass | $m_{\mathrm{e}}$ | 9.109 383 701 5(28) | $\times 10^{-31}$ | kg |
| Neutron mass | $m_{\mathrm{n}}$ | 1.674 927 498 04(95) | $\times 10^{-27}$ | kg |
| Proton mass | $m_{\mathrm{p}}$ | 1.672 621 923 69(51) | $\times 10^{-27}$ | kg |
| Standard acceleration of gravity | $g_{\mathrm{n}}$ | 9.806 65 | | $\mathrm{m \cdot s^{-2}}$ |

## Long problems

Duration: 150 minutes
Marks: 48 points ($3 \times 16$)

Start each problem on a new sheet in order to ease the correction.

*General hint: The problems consist of partially independent problem parts, so if you get stuck, it is a good idea to read further ahead and to continue with an easier part.*

### Long problem 1.1: Twin paradox *(16 points)*

Special relativity introduced the concept of time dilation. One of the most common misconceptions about this concept leads to the famous twin paradox. In this exercise, you will solve this paradox step by step with the help of spacetime diagrams.

#### Part A. Spacetime diagrams *(2.5 points)*

Spacetime diagrams are a very useful tool to work with problems in special relativity. In such diagrams, time multiplied by the speed of light $ct$ is drawn on the vertical axis, while position $x$ is drawn on the horizontal axis. Same units are used on the horizontal and vertical axes.

**i. (1 pt)** Draw on a spacetime diagram the two lines corresponding to a particle travelling at the speed of light $c$ and going through the origin. The union of these lines is called the light cone.

**ii. (0.5 pts)** Alice and Bob are twins. On their twentieth birthday, Alice decides to get on a rocket to travel to a star that is $x_1 = 1\,\mathrm{ly}$ (light-year) away at a speed $v = \frac{c}{2}$ and, once the star is reached, to come back on Earth at the same speed in the opposite direction. Bob on the other hand stays on the Earth, waiting for Alice's return. Let us see how time dilation affects the ageing of both Alice and Bob. For this, start by drawing the trajectory of Bob from time $t_0 = 0$ to time $t_2$ (the time when Alice and Bob meet again) while he stays at position $x = 0$ during the whole trip.

**iii. (1 pt)** Draw on the same diagram the trajectory of Alice. We associate coordinate time $t_1$ to the event of Alice reaching the star and changing her direction.

#### Part B. Time dilation *(3.5 points)*

You may have heard that in special relativity, time dilation can occur when a system is moving with respect to another. In a naive way, Bob who stays on Earth would expect Alice to be younger than him at time $t_2$, because from his perspective, Alice was moving and was thus subject to time dilation. However, Alice could observe Bob's motion from her perspective and, by the same reasoning, she would think that Bob would be younger when they meet again. This is the twin paradox. The goal of this part is to solve this paradox.

**i. (0.5 pts)** Write down the difference in proper time squared $(\Delta\tau)^2$ as a function of $\Delta t$ and $\Delta x$. This expression can be thought of as the Pythagorean theorem in spacetime. A system's proper time refers to time as measured inside the said system.

**ii. (0.5 pts)** Compute how much time has passed for Bob when he meets Alice again after she comes back from her trip.
*Hint: you might want to use your diagram to solve the problem geometrically.*

**iii. (1.5 pts)** Compute how much time has passed for Alice during her whole trip.

**iv. (1 pt)** Describe qualitatively the physical difference between Alice and Bob that explains your result of the two preceding questions.

#### Part C. Alice's perspective *(5.5 points)*

Let us get a deeper understanding of the situation by considering Alice's coordinates.

**i. (1 pt)** In the above, we implicitly considered Bob's system of coordinates $(ct, x)$. Write down Alice's system of coordinates $(ct', x')$ as a function of $t$, $x$, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ and $\beta = v/c$.

**ii. (2 pts)** Draw on your diagram the simultaneity line for Alice as she reaches the star, i.e. the line with $t' = \text{const.}$ passing through the event of Alice reaching the star. Determine how old Bob is from the point of view of Alice when she reaches the star.

**iii. (1.5 pts)** Let us write the coordinate system of Alice after she reverses her velocity as $(ct'', x'')$. Draw on your diagram the simultaneity line for Alice just after she changed her direction. Determine with the help of your diagram how old Bob is from the point of view of Alice just after turning around. *Hint: How do you expect the slope of the simultaneity line in the $(t'', x'')$ frame to be related to the one in the $(t', x')$ frame?*

**iv. (1 pt)** Compare your result to the one just before reaching the star. Would you expect this result? Why?

### Part D. Doppler effect *(4.5 points)*

Now let us assume that Alice told Bob that she would point a light beam with wavelength $\lambda = 500\,\mathrm{nm}$ in the direction of Earth during her whole journey.

**i. (1 pt)** Compute the wavelength of the signal sent by Alice before she reaches the star as observed by Bob.

**ii. (0.5 pts)** Compute the wavelength of the signal sent by Alice after she changes her direction as observed by Bob.

**iii. (1 pt)** Determine the time $t_c$ at which Bob observes a change in the colour of the signal. *Hint: Consider using your diagram.*

**iv. (2 pts)** How can Bob use his measurement of $t_c$ and of the wavelengths before and after the shift to predict the age of Alice as she comes back to Earth? Compare your result with the one from above.

## Long problem 1.2: Balloon *(16 points)*

The goal of this task is to calculate the flight altitude of a helium-filled weather balloon. To do this, we first consider the physics of the atmosphere. Assume that the atmosphere consists of pure nitrogen $^{14}N$, which can be assumed to be an ideal gas of nitrogen molecules $N_2$.

### Part A. Atmospheric model *(6.5 points)*

**i. (1 pt)** Which forces are relevant for the description of a static air column?

**ii. (3.5 pts)** Assume that we have a temperature profile $T(z)$ in the atmosphere. Now use the ideal gas equation to derive a differential equation for the density $\rho$.

**iii. (2 pts)** We assume that the temperature profile can be described by the function $T(z) = T_0\,(1 + \alpha z)$ with $T_0$ the temperature at sea level $z = 0$. Solve the differential equation with the ansatz

$$\rho(z) = A\,(1 + \alpha z)^{-B}$$

and determine the constants $A$ and $B$ as a function of $\rho_0$ (density at sea level), $T_0$, the molar mass $m$ and $\alpha$. We will continue with this ansatz in the following tasks.

### Part B. Ascension *(9.5 points)*

Now suppose we are at sea level $z = 0$. Assume the air pressure is 1.0 bar and it is sunny 25 °C. We now pump helium $^4$He into a balloon with a net mass of 3.0 g until it has a volume of 5.0 L. This creates an overpressure of 5000 Pa in the balloon. We assume that the overpressure remains constant during the ascent. You may additionally assume that the helium is an ideal gas.

**i. (1 pt)** Calculate the air density at sea level and give the result in $\mathrm{g \cdot m^{-3}}$.

**ii. (1 pt)** Describe briefly in words why helium, as an ideal gas, has a lower density than nitrogen. Which parameter is decisive?

**iii. (2.5 pts)** What is the mass of the balloon and what is the force acting on it at sea level? Give both the formula and the numerical value. Assume here and in the following that the helium in the balloon has the same temperature as the surrounding nitrogen in the atmosphere.

**iv. (4 pts)** Now calculate the height the balloon can reach. Use that $\alpha = -0.017\,\mathrm{km^{-1}}$.

**v. (1 pt)** Even if the overpressure at sea level does not make much difference, it is still important. How high could the balloon theoretically fly if we did not take the overpressure into account? A short explanation without calculation is enough.

## Long problem 1.3: OSA Jupiter Mission *(16 points)*

The OSA (Olympiad Space Agency) wants to start its first interplanetary mission. For this, they need some preliminary calculations from your side to ensure mission success.

### Part A. Celestial Dynamics *(1.5 points)*

The OSA mission committee has decided that Jupiter will be the destination of their probe. They need you to calculate some properties of their destination.

**i. (1.5 pts)** Given the duration of a Jupiter year ($T_{\text{Jupiter}} = 4333\,\text{d}$) and the masses of the Jupiter-Sun system,

$$m_{\text{Jupiter}} = 1.899 \times 10^{27}\,\text{kg},$$

$$M_{\text{Sun}} = 1.9884 \times 10^{30}\,\text{kg},$$

calculate the radius $R_{\text{Jupiter}}$ of Jupiter's orbit. You can assume the Sun to be much heavier than Jupiter and the orbits to be circular.

### Part B. Sun-tides on Jupiter *(4 points)*

OSA scientists have postulated the existence of tides on Jupiter, which, according to them, could lead to drastic drops in accuracy for Jupiter-based horoscopes. To verify (or disprove) this hypothesis you are tasked to do some calculations.

**i. (1 pt)** Calculate the surface gravity of Jupiter (assuming for now that the gravity of other bodies and the rotation around the Sun has no effect). Let $r_{\text{Jupiter}} = 69\,911\,\text{km}$ be the radius of Jupiter. You can assume Jupiter to be a perfect sphere.

**ii. (2 pts)** Now consider a reference frame rotating with Jupiter around the Sun. Calculate the total acceleration at the Sun side of Jupiter in this reference frame, neglecting the gravitational pull of Jupiter itself. Do the same for the dark side.

**iii. (1 pt)** Explain how the above calculations show the existence of (solar) tides on Jupiter. Do you think this effect is noticeable on the surface of Jupiter?

### Part C. JWST 2.0 *(3.5 points)*

OSA scientists want to build a space telescope that is shielded from the Sun's radiation by Jupiter. For this they will place the telescope at the so-called Lagrange point 2 (L2).

**i. (0.5 pts)** Lagrange points are points in a two-body system (in this case Jupiter and the Sun), where the net force acting on a third body is zero in the rotating reference frame. Qualitatively explain why such points can exist.

**ii. (1 pt)** Using the question about the tides, write down an equation for the net acceleration experienced by a body that is placed at a distance $d$ behind (on the dark side) Jupiter in the rotating reference frame. What conditions need to hold at the Lagrange points? You do not need to solve this equation.

**iii. (2 pts)** Explain why typically no asteroids reside at L2. Show this explicitly using your previous results.

### Part D. Getting to Jupiter *(7 points)*

Thanks to your calculations, the OSA now feels confident that they can actually put a satellite at the desired location. Now they "only" need to bring it there. For this, OSA scientists proposed the following multi-stage trajectory:

1. Leave the Earth's gravitational influence by traveling along the Earth's orbit, until we are in circular orbit around the Sun.
2. Using a strong burn, change our orbit to be so elliptical that the farthest point of the ellipse meets Jupiter's orbit.
3. Upon reaching Jupiter's orbit, quickly accelerate again to correct the orbit into Jupiter's circular orbit.

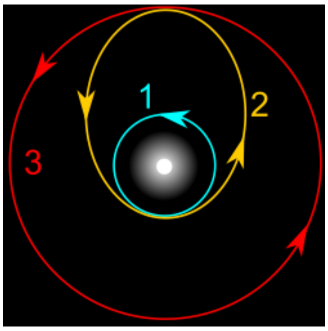



Figure D.1: The planned trajectory

**i. (1 pt)** Calculate the minimal velocity $v_{\text{escape}}$ needed to leave the Earth's gravitational influence (the speed at which an object traveling radially outwards will never fall back onto the Earth). You may neglect all other planetary bodies. You may neglect the Earth's rotation. $r_{\text{Earth}} = 6371\,\text{km}$, $R_{\text{Earth}} = 1.496 \times 10^{8}\,\text{km}$, $m_{\text{Earth}} = 5.972 \times 10^{24}\,\text{kg}$. We will launch the satellite using an Atlas V rocket, which weighs $m_{\text{rocket}} = 587\,\text{t}$ at start, including fuel.

**ii. (3.5 pts)** We assume now that we are in an orbit around the Sun. Calculate the (instantaneous) change in velocity needed to change into the transfer orbit.
*Hint: use the conservation of energy and angular momentum.*

**iii. (1 pt)** Calculate the change in velocity to change to Jupiter's orbit (you may ignore the gravitational pull of Jupiter).

**iv. (1.5 pts)** Using the energy density of rocket fuel ($50\,\text{MJ} \cdot \text{kg}^{-1}$) calculate the amount of fuel needed to accomplish all three manoeuvres. Assume the mass of the rocket (including fuel) to be constant. Is this assumption justified?

# Long problems: solutions

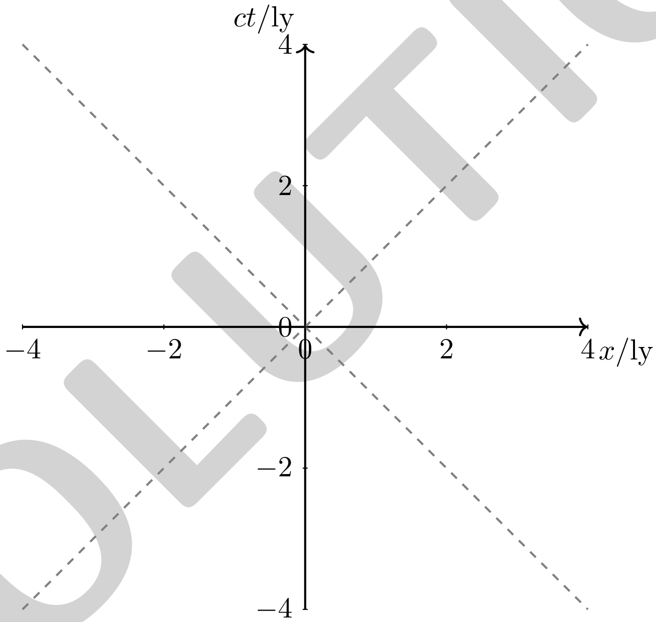
## Long problem 1.1: Twin paradox *16*

**Special relativity introduced the concept of time dilation. One of the most common misconceptions about this concept leads to the famous twin paradox. In this exercise, you will solve this paradox step by step with the help of spacetime diagrams.**

### Part A. Spacetime diagrams *2.5*

**Spacetime diagrams are a very useful tool to work with problems in special relativity. In such diagrams, time multiplied by the speed of light $ct$ is drawn on the vertical axis, while position $x$ is drawn on the horizontal axis. Same units are used on the horizontal and vertical axes.**

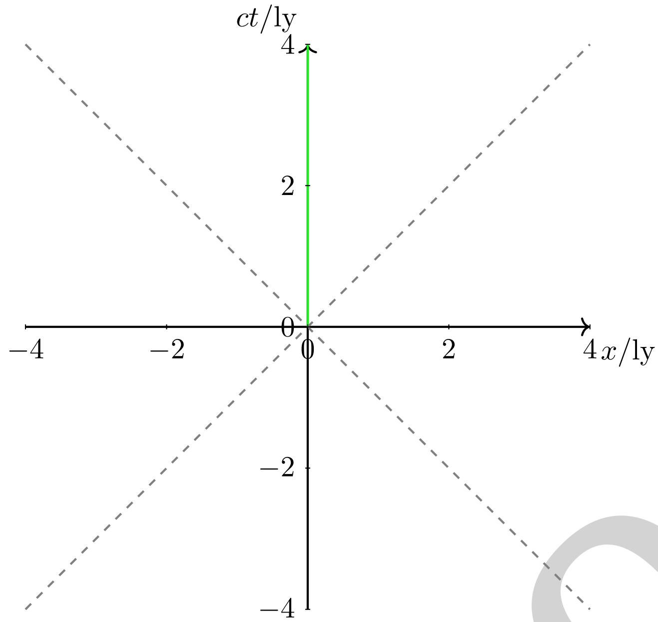
**i. Draw on a spacetime diagram the two lines corresponding to a particle travelling at the speed of light $c$ and going through the origin. The union of these lines is called the light cone.** *1*





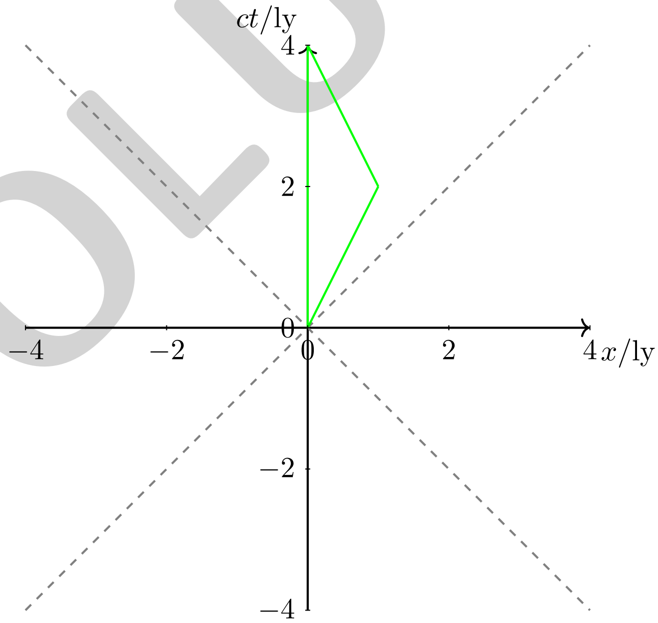

0.5 points for each line. 1

**ii. Alice and Bob are twins. On their twentieth birthday, Alice decides to get on a rocket to travel to a star that is $x_1 = 1\,\text{ly}$ (light-year) away at a speed $v = \frac{c}{2}$ and, once the star is reached, to come back on Earth at the same speed in the opposite direction. Bob on the other hand stays on the Earth, waiting for Alice's return. Let us see how time dilation affects the ageing of both Alice and Bob. For this, start by drawing the trajectory of Bob from time $t_0 = 0$ to time $t_2$ (the time when Alice and Bob meet again) while he stays at position $x = 0$ during the whole trip.** *0.5*




0.5

**iii. Draw on the same diagram the trajectory of Alice. We associate coordinate time $t_1$ to the event of Alice reaching the star and changing her direction.** ***1***




0.5 points for each segment. 1

## Part B. Time dilation *3.5*

**You may have heard that in special relativity, time dilation can occur when a system is moving with respect to another. In a naive way, Bob who stays on Earth would expect Alice to be younger than him at time $t_2$, because from his perspective, Alice was moving and was thus subject to time dilation. However, Alice could observe Bob's motion from her perspective and, by the same reasoning, she would think that Bob would be younger when they meet again. This is the twin paradox. The goal of this part is to solve this paradox.**

**i. Write down the difference in proper time squared $(\Delta\tau)^2$ as a function of $\Delta t$ and $\Delta x$. This expression can be thought of as the Pythagorean theorem in spacetime. A system's proper time refers to time as measured inside the said system.** ***0.5***

We have $(\Delta\tau)^2 = (\Delta t)^2 - \frac{(\Delta x)^2}{c^2}$. 0.5

**ii. Compute how much time has passed for Bob when he meets Alice again after she comes back from her trip.**
***Hint: you might want to use your diagram to solve the problem geometrically.*** ***0.5***

As Alice has speed $c/2$ and does a round trip to a star 1 ly away, we have $t_2 = 4\,\mathrm{a}$. As for Bob $\Delta x = 0$, we obtain $\Delta\tau_\mathrm{B} = 4\,\mathrm{a}$. 0.5

**iii. Compute how much time has passed for Alice during her whole trip.** ***1.5***

The situation for Alice is different, as she gets into another intertial frame after turning around. One thus has to add the contributions to proper time of the two parts of her trip separately. 0.5

One thus gets $\Delta\tau_\mathrm{A} = \Delta\tau_1 + \Delta\tau_2$, where $\Delta\tau_1 = \Delta\tau_2 = \sqrt{\left(\frac{t_2}{2}\right)^2 - x_1^2/c^2} = 3\,\mathrm{a}$ such that $\Delta\tau_\mathrm{A} = 2\sqrt{3}\mathrm{a} \approx 3.5\,\mathrm{a}$. 1

**iv. Describe qualitatively the physical difference between Alice and Bob that explains your result of the two preceding questions.** ***1***

The main difference is that while Bob remains an inertial observer, Alice jumps from one inertial frame to another at the moment she turns around. 1

## Part C. Alice's perspective *5.5*

**Let us get a deeper understanding of the situation by considering Alice's coordinates.**

**i. In the above, we implicitly considered Bob's system of coordinates $(ct, x)$. Write down Alice's system of coordinates $(ct', x')$ as a function of $t$, $x$, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ and $\beta = v/c$.** ***1***

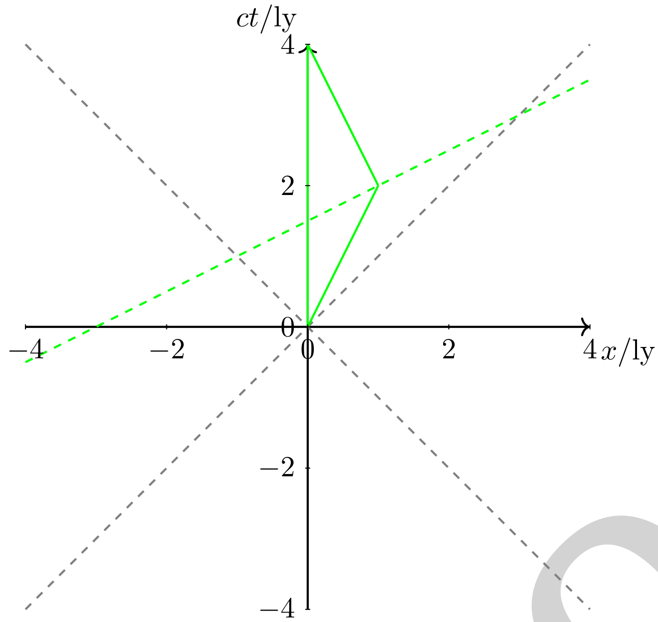
The Lorentz transformation is given by (0.5 points for each coordinate)

$$\begin{cases} ct' = \gamma ct - \beta\gamma x \\ x' = -\beta\gamma ct + \gamma x \end{cases}.$$

1

**ii. Draw on your diagram the simultaneity line for Alice as she reaches the star, i.e. the line with $t' =$ const. passing through the event of Alice reaching the star. Determine how old Bob is from the point of view of Alice when she reaches the star.** ***2***

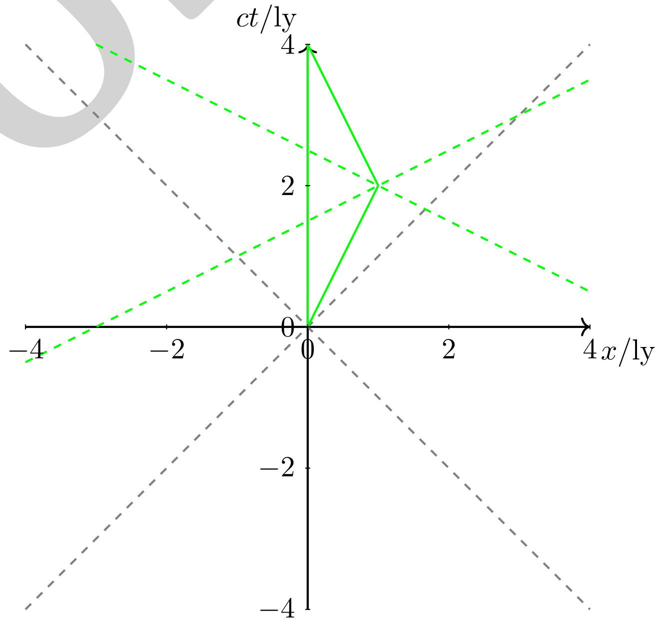
Isolating $t$ in the Lorentz transformation, one sees that the coefficient of $x$ is $\beta = 1/2$. Thus, we want to draw the line with slope 1/2 passing through (1 ly, 2 ly).




1

The intersection with the $ct$-axis is given by $t = 1.5\,\mathrm{ly}$, so Bob would be $21.5\,\mathrm{a}$ old as Alice reaches the star, according to Alice. 1

**iii. Let us write the coordinate system of Alice after she reverses her velocity as $(ct'', x'')$. Draw on your diagram the simultaneity line for Alice just after she changed her direction. Determine with the help of your diagram how old Bob is from the point of view of Alice just after turning around.**
***Hint: How do you expect the slope of the simultaneity line in the $(t'', x'')$ frame to be related to the one in the $(t', x')$ frame?*** ***1.5***



0.5

The result can be seen on the diagram by noticing that the slope in the second inertial frame is opposite to the one in the first one because the velocity is opposite. Then, one can see from the diagram that

$$t_{B''} = t_1 + (t_1 - t_{B'}) = 2t_1 - t_{B'} = 2.5\,\mathrm{ly}.$$

Thus, just after Alice turns around, Bob would be 22.5 years old. 1

**iv. Compare your result to the one just before reaching the star. Would you expect this result? Why?** ***1***

From the point of view of Alice, once she changes her velocity, the age of Bob in her system of coordinates increases by one year almost "instantly". This is very counter-intuitive from a classical mechanics point of view, but we know from the previous section that something "special" happens because Alice changes her direction. This is reflected in this last (maybe surprising) result. 1

## Part D. Doppler effect *4.5*

**Now let us assume that Alice told Bob that she would point a light beam with wavelength $\lambda = 500\,\mathrm{nm}$ in the direction of Earth during her whole journey.**

**i. Compute the wavelength of the signal sent by Alice before she reaches the star as observed by Bob.** ***1***

*The relativistic Doppler effect can be written as*

$$\lambda_{obs} = \lambda_{em}\sqrt{\frac{1+v/c}{1-v/c}}$$

*where $v > 0$ for the emitter getting further away from the observer, such that we get $\lambda_{obs} = 500\,\mathrm{nm}\cdot\sqrt{3} \approx 866\,\mathrm{nm}$.*

Correct formula. 0.5

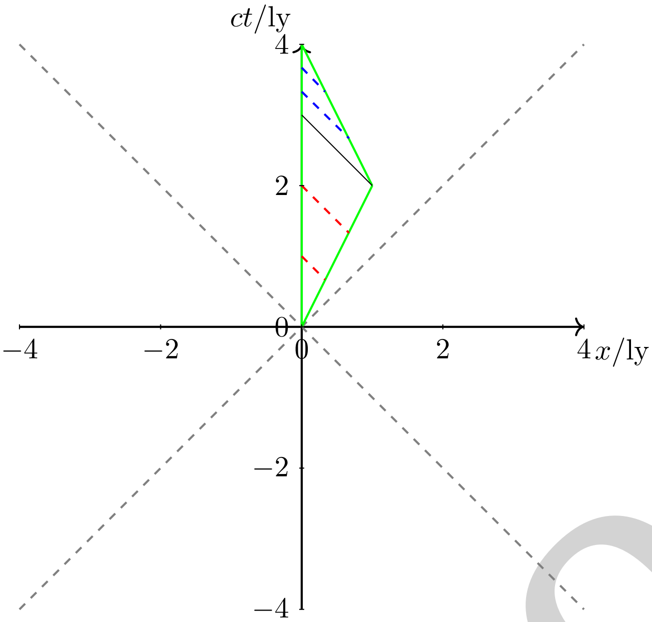
Numerical result. 0.5

**ii. Compute the wavelength of the signal sent by Alice after she changes her direction as observed by Bob.** ***0.5***

Using the same formula as before, but with a reversed sign, we obtain $\lambda_{\mathrm{obs}} = 500\,\mathrm{nm}/\sqrt{3} \approx 289\,\mathrm{nm}$. 0.5

**iii. Determine the time $t_c$ at which Bob observes a change in the colour of the signal.**
***Hint: Consider using your diagram.*** ***1***

*Graphically, one can draw a line with slope $-1$ starting at the event where Alice reaches the star, and find the intersection with the ct-axis. One finds $t_c = 3\,\mathrm{a}$ (see black line in the following diagram (the participants are not asked to draw this)).*






| | |
|---|---|
| Line with slope -1. | 0.5 |
| Correct numerical answer. | 0.5 |

**iv. How can Bob use his measurement of $t_c$ and of the wavelengths before and after the shift to predict the age of Alice as she comes back to Earth? Compare your result with the one from above.** *2*

Bob measured $t_c$ as well as the wavelength of the beam before and after the change in wavelength. Comparing these wavelengths with the original wavelength of 500 nm, he can deduce how fast time passes for Alice for both cases of redshift and blueshift. In the first 3 years, we have $\lambda_{obs}/\lambda_{em} = \sqrt{3}$ and in the last year, we have $\lambda_{obs}/\lambda_{em} = 1/\sqrt{3}$, so one can compute (using that the wavelength is proportional to the period)

$$\Delta\tau_A = 3/\sqrt{3}\mathrm{a} + 1\sqrt{3}\mathrm{a} = 2\sqrt{3}\mathrm{a} \approx 3.5\,\mathrm{a}.$$

This is the same result as before. Alice would thus come back as a 23.5 a old. 2

## Long problem 1.2: Balloon — 16

**The goal of this task is to calculate the flight altitude of a helium-filled weather balloon. To do this, we first consider the physics of the atmosphere. Assume that the atmosphere consists of pure nitrogen $^{14}N$, which can be assumed to be an ideal gas of nitrogen molecules $N_2$.**

### Part A. Atmospheric model — 6.5

**i. Which forces are relevant for the description of a static air column?** *1*

The weight and the force caused by the gas pressure always apply. (0.5 points each) 1

**ii. Assume that we have a temperature profile $T(z)$ in the atmosphere. Now use the ideal gas equation to derive a differential equation for the density $\rho$.** *3.5*

We start with the hydrostatic pressure on an air parcel of height $\Delta z$

$$\Delta z \rho g = -\Delta p.$$

0.5

For an infinitesimal small height $\Delta z$ we get

$$\rho g = -\frac{\partial p}{\partial z}.$$

0.5

The ideal gas law allows to express the pressure in dependence of $\rho$: $p = \frac{\rho R T}{m}$, where $m$ is the molar mass of the gas. 1

We use the product rule to get an expression for $p$

$$\frac{\partial p}{\partial z} = \frac{\partial \rho}{\partial z}\frac{RT}{m} + \frac{\partial T}{\partial z}\frac{\rho R}{m}.$$

1

We get the differential equation

$$\frac{\partial \rho}{\partial z} + \left(\frac{mg}{RT} + \frac{1}{T}\frac{\partial T}{\partial z}\right)\rho = 0.$$

0.5

**iii. We assume that the temperature profile can be described by the function $T(z) = T_0\,(1+\alpha z)$ with $T_0$ the temperature at sea level $z = 0$. Solve the differential equation with the ansatz**

$$\rho(z) = A\,(1+\alpha z)^{-B}$$

**and determine the constants $A$ and $B$ as a function of $\rho_0$ (density at sea level), $T_0$, the molar mass $m$ and $\alpha$. We will continue with this ansatz in the following tasks.** *2*

We plug in the Ansatz into the differential equation

$$-AB\alpha\,(1+\alpha z)^{-B-1} + \left(\frac{mg}{R} + T_0\alpha\right) A\frac{(1+\alpha z)^{-B-1}}{T_0} = 0,$$

which gives

$$-B\alpha + \left(\frac{mg}{RT_0} + \alpha\right) = 0.$$

1

This means $B = \left(\frac{mg}{R\alpha T_0} + 1\right)$. 0.5

The constant $A = \rho_0$ is obtained from the initial condition at $z = 0$. 0.5

## Part B. Ascension *9.5*

**Now suppose we are at sea level $z = 0$. Assume the air pressure is $1.0\,\mathrm{bar}$ and it is sunny $25\,^\circ\mathrm{C}$. We now pump helium $^4$He into a balloon with a net mass of $3.0\,\mathrm{g}$ until it has a volume of $5.0\,\mathrm{L}$. This creates an overpressure of $5000\,\mathrm{Pa}$ in the balloon. We assume that the overpressure remains constant during the ascent. You may additionally assume that the helium is an ideal gas.**

**i. Calculate the air density at sea level and give the result in $\mathrm{g \cdot m^{-3}}$.** *1*

We get, using the ideal gas equation,

$$\rho = \frac{pm}{RT} = 1130\,\mathrm{g \cdot m^{-3}}.$$

(0.5 points for equation, 0.5 for correct numerical value) 1

**ii. Describe briefly in words why helium, as an ideal gas, has a lower density than nitrogen. Which parameter is decisive?** *1*

Because both are ideal gases, they have the same molar density. The difference lies in the molar mass. 1

**iii. What is the mass of the balloon and what is the force acting on it at sea level? Give both the formula and the numerical value. Assume here and in the following that the helium in the balloon has the same temperature as the surrounding nitrogen in the atmosphere.** *2.5*

We have $M_{\mathrm{balloon}} = M_{\mathrm{He}} + M_{\mathrm{rubber}} = \frac{(p_0+\Delta p)m_{\mathrm{He}}}{RT}V + M_{\mathrm{rubber}} = 3.8\,\mathrm{g}$. (0.5 points for equation, 0.5 for correct numerical value) 1

The displaced gas has a mass of $M_{\mathrm{gas}} = \frac{p_0 m_{\mathrm{N}}}{RT}V = 5.6\,\mathrm{g}$. (Numerical value not needed) 0.5

The buoyancy applied on the balloon is therefore $(M_{\mathrm{gas}} - M_{\mathrm{balloon}})\,g = 0.018\,\mathrm{N}$. (0.5 points for equation, 0.5 for correct numerical value) 1

**iv. Now calculate the height the balloon can reach. Use that $\alpha = -0.017\,\mathrm{km}^{-1}$.** *4*

At the maximal height the mass of the displaced gas is equal to the mass of the balloon, which doesn't change during flight.

$$M_{\mathrm{gas}}(z) = M_{\mathrm{ballon}}.$$

0.5

The mass of the displaced nitrogen at height $z$ is

$$M_{\mathrm{gas}}(z) = \frac{p_{\mathrm{N}}(z)\,m_{\mathrm{N}}}{RT}V.$$

0.5

The volume and temperature is the same as for the helium inside the ballon, meaning by the ideal gas law

$$\frac{n_{\mathrm{He}}}{(p(z)+\Delta p)} = \frac{V}{RT}.$$

And we obtain

$$M_{\mathrm{gas}}(z) = \frac{p(z)\, m_{\mathrm{N}} n_{\mathrm{He}}}{p(z)+\Delta p}.$$

0.5

The equilibrium equation can now be solved for $p(z)$

$$p(z) = \frac{M_{\mathrm{balloon}}\Delta p}{m_{\mathrm{N}} n_{\mathrm{He}} - M_{\mathrm{balloon}}}.$$

0.5

*Note that we can express* $m_{\mathrm{N}} n_{\mathrm{He}}$ *as*

$$m_{\mathrm{N}} n_{\mathrm{He}} = m_{\mathrm{N}} n_{\mathrm{N}}(0)\, \frac{p(0)+\Delta p}{p(0)} = M_{gas}(0)\, \frac{p(0)+\Delta p}{p(0)}.$$

The pressure on height $z$ can be obtained by integration

$$p(z) = p_0\,(1+\alpha z)^{-\frac{m_{\mathrm{N}} g}{R\alpha T_0}}.$$

1

Solving for height $H$ gives

$$H = \left(\left(\frac{\Delta p}{p_0}\frac{M_{\mathrm{balloon}}}{m_{\mathrm{N}} n_{\mathrm{He}} - M_{\mathrm{balloon}}}\right)^{-\frac{R\alpha T_0}{m_{\mathrm{N}} g}} - 1\right)\frac{1}{\alpha} = 18\,\mathrm{km}.$$

(0.5 points for equation, 0.5 for correct numerical value) 1

**v. Even if the overpressure at sea level does not make much difference, it is still important. How high could the balloon theoretically fly if we did not take the overpressure into account? A short explanation without calculation is enough.** ***1***

Because both helium and nitrogen are ideal gases, they behave identically. This means in particular that they expand the same. The nitrogen displaced by the balloon therefore always weighs the same, whether at sea level or in the upper atmosphere. The balloon theoretically rises infinitely high. 1

## Long problem 1.3: OSA Jupiter Mission *16*

**The OSA (Olympiad Space Agency) wants to start its first interplanetary mission. For this, they need some preliminary calculations from your side to ensure mission success.**

### Part A. Celestial Dynamics *1.5*

**The OSA mission committee has decided that Jupiter will be the destination of their probe. They need you to calculate some properties of their destination.**

**i. Given the duration of a Jupiter year ($T_{\mathbf{Jupiter}} = 4333\,\mathrm{d}$) and the masses of the Jupiter-Sun system,**

$$m_{\mathbf{Jupiter}} = 1.899 \times 10^{27}\,\mathrm{kg},$$

$$M_{\mathbf{Sun}} = 1.9884 \times 10^{30}\,\mathrm{kg},$$

**calculate the radius $R_{\mathbf{Jupiter}}$ of Jupiter's orbit. You can assume the Sun to be much heavier than Jupiter and the orbits to be circular.** ***1.5***

*Assuming that Jupiter's orbit around the Sun is circular, the semi-major and semi-minor axes of the "degenerate" elliptical trajectory would coincide with the radius $R_{Jupiter}$ of the corresponding circular trajectory.*

Using the 3rd Kepler's law $\frac{R^3_{\text{Jupiter}}}{T^2_{\text{Jupiter}}} = \frac{G(M_{\text{Sun}}+m_{\text{Jupiter}})}{4\pi^2} \approx \frac{GM_{\text{Sun}}}{4\pi^2}$ or alternatively by considering the equilibrium between centrifugal and gravitational forces,

$$\frac{mv^2}{R_{\text{Jupiter}}} = \frac{GMm}{R^2_{\text{Jupiter}}}$$

with the velocity $v$ of an object in nearly circular orbit, that can be approximated as

$$v = \frac{2\pi R_{\text{Jupiter}}}{T},$$

0.5

we get:

$$R_{\text{Jupiter}} = \sqrt[3]{\frac{GM_{\text{Sun}}T^2_{\text{Jupiter}}}{4\pi^2}}.$$

0.5

The numerical value is

$$R_{\text{Jupiter}} = 7.78 \times 10^{11}\,\mathrm{m} = 7.78 \times 10^{8}\,\mathrm{km}.$$

0.5

### Part B. Sun-tides on Jupiter *4*

**OSA scientists have postulated the existence of tides on Jupiter, which, according to them, could lead to drastic drops in accuracy for Jupiter-based horoscopes. To verify (or disprove) this hypothesis you are tasked to do some calculations.**

**i. Calculate the surface gravity of Jupiter (assuming for now that the gravity of other bodies and the rotation around the Sun has no effect). Let $r_{\mathbf{Jupiter}} = 69\,911\,\mathrm{km}$ be the radius of Jupiter. You can assume Jupiter to be a perfect sphere.** ***1***

According to the 2nd Newton's law, mass times acceleration is equal to the sum of all the forces:

$$m\vec{a} = \sum_i \vec{F}_i.$$

0.25

We assumed that only the gravitional pull has an effect and therefore:

$$ma = G\frac{m m_{\text{Jupiter}}}{r_{\text{Jupiter}}^2}$$

0.5

and find:

$$a_g = G\frac{m_{\text{Jupiter}}}{r^2} = 25.9\,\text{m}\cdot\text{s}^{-2}.$$

0.25

**ii. Now consider a reference frame rotating with Jupiter around the Sun. Calculate the total acceleration at the Sun side of Jupiter in this reference frame, neglecting the gravitational pull of Jupiter itself. Do the same for the dark side.** *2*

The gravitational pull of the Sun and the centrifugal force have the same direction but opposite signs, so that by Newton's law the magnitude of the resulting acceleration is given by

$$a_{\text{tot}} = -\frac{GM_{\text{Sun}}}{R^2} + \omega^2 R$$

with $\omega = 2\pi R/T$. (0.5 points for each contribution) 1

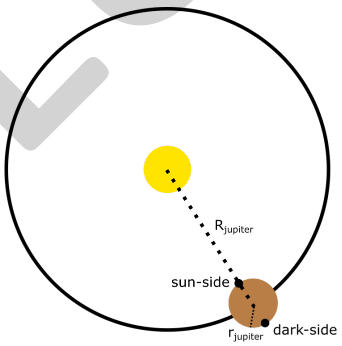



Figure B.1: Sketch

As it is clear from a sketch of Jupiter's orbit, and assuming that Jupiter's orbit around the Sun is circular and that both Jupiter and the Sun are perfect spheres, we have that

$$R_{\text{Sun}} = R_{\text{Jupiter}} - r_{\text{Jupiter}},$$

$$R_{\text{dark}} = R_{\text{Jupiter}} + r_{\text{Jupiter}},$$

0.5

giving an acceleration of

$$a_{\text{tot, Sun}} = 5.909 \times 10^{-8}\,\mathrm{m \cdot s^{-2}},$$

$$a_{\text{tot, dark}} = -5.908 \times 10^{-8}\,\mathrm{m \cdot s^{-2}}.$$

(0.25 points for each correct numerical result) 0.5

**iii. Explain how the above calculations show the existence of (solar) tides on Jupiter. Do you think this effect is noticeable on the surface of Jupiter?** ***1***

At the center of Jupiter this acceleration is zero (condition for a stable orbit). Therefore we have a difference in acceleration between the center and the Sun/dark points causes a slight relative upwards acceleration (of about the same magnitude on both the Sun and the dark side) as compared to points located on the orbit. 0.5

This could cause slight tides to form, but the effect is very weak, so we probably couldn't notice it on the surface of Jupiter. 0.5

## Part C. JWST 2.0 *3.5*

**OSA scientists want to build a space telescope that is shielded from the Sun's radiation by Jupiter. For this they will place the telescope at the so-called Lagrange point 2 (L2).**

**i. Lagrange points are points in a two-body system (in this case Jupiter and the Sun), where the net force acting on a third body is zero in the rotating reference frame. Qualitatively explain why such points can exist.** ***0.5***

Consider the reference frame rotating with Jupiter around the Sun as in part B. The centrifugal force increases as we travel outwards from Jupiter, while gravity increases in the opposite direction. Thus there is a point where both need to be equal, which results in (some of) the Lagrange points. 0.5

**ii. Using the question about the tides, write down an equation for the net acceleration experienced by a body that is placed at a distance $d$ behind (on the dark side) Jupiter in the rotating reference frame. What conditions need to hold at the Lagrange points? You do not need to solve this equation.** ***1***

The acceleration is due to three forces:

1. Jupiter's gravity: $\frac{Gm_{\text{Jupiter}}}{d^2}$,
2. the Sun's gravity: $\frac{GM_{\text{Sun}}}{(d+R_{\text{Jupiter}})^2}$,
3. the centrifugal acceleration: $-\omega^2\,(d + R_{\text{Jupiter}})$.

We can use Newton's law as in part B to get the acceleration

$$a = \frac{Gm_{\text{Jupiter}}}{d^2} + \frac{GM_{\text{Sun}}}{(d+R_{\text{Jupiter}})^2} - \omega^2\,(d + R_{\text{Jupiter}})\,.$$

(If the centrifugal force and gravity of Jupiter are stated here but not in Bii., the points might be awarded in Bii.) 0.5

The total acceleration (in the rotating frame of reference) needs to be zero. 0.5

**iii. Explain why typically no asteroids reside at L2. Show this explicitly using your previous results.** *2*

This question is asking for the stability of the L2 point. For this we need to check if some small perturbation, which displaces the asteroid by $\Delta d$ away from the L2 point, accelerates the asteroid away from the L2 point. Therefore we plug $d + \Delta d$ into the formula for the acceleration

$$a' = \frac{Gm_{\text{Jupiter}}}{(d+\Delta d)^2} + \frac{GM_{\text{Sun}}}{(d+R_{\text{Jupiter}}+\Delta d)^2} - \omega^2 \left(d + R_{\text{Jupiter}} + \Delta d\right).$$

0.5

The first two terms get smaller as $\Delta d$ increases. So does the last term due to the minus sign. In total we get $a' < 0$, which corresponds to an acceleration away from Jupiter. 0.5

The first two terms get larger as $\Delta d$ decreases. So does the last term due to the minus sign. In total we get $a' > 0$, which corresponds to an acceleration towards Jupiter. 0.5

Thus this point is unstable. 0.5

## Part D. Getting to Jupiter *7*

**Thanks to your calculations, the OSA now feels confident that they can actually put a satellite at the desired location. Now they "only" need to bring it there. For this, OSA scientists proposed the following multi-stage trajectory:**

1. **Leave the Earth's gravitational influence by traveling along the Earth's orbit, until we are in circular orbit around the Sun.**
2. **Using a strong burn, change our orbit to be so elliptical that the farthest point of the ellipse meets Jupiter's orbit.**
3. **Upon reaching Jupiter's orbit, quickly accelerate again to correct the orbit into Jupiter's circular orbit.**

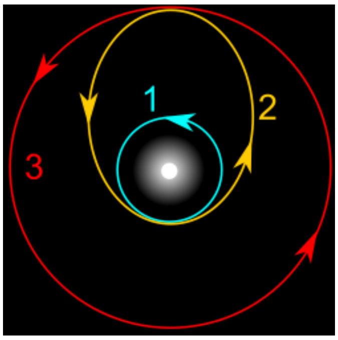



Figure D.1: The planned trajectory

**i. Calculate the minimal velocity $v_{\mathbf{escape}}$ needed to leave the Earth's gravitational influence (the speed at which an object traveling radially outwards will never fall back onto the Earth). You may neglect all other planetary bodies. You may neglect the Earth's rotation. $r_{\mathbf{Earth}} = 6371\,\mathrm{km}$, $R_{\mathbf{Earth}} = 1.496 \times 10^8\,\mathrm{km}$, $m_{\mathbf{Earth}} = 5.972 \times 10^{24}\,\mathrm{kg}$. We will launch the satellite using an Atlas V rocket, which weighs $m_{\mathbf{rocket}} = 587\,\mathrm{t}$ at start, including fuel.** ***1***

In order to escape the Earth's gravitational field, the potential barrier needs to be overcome, meaning that the initial kinetic energy has to be equal to the potential energy on the Earth's surface.

$$E_{\mathrm{kin}} = E_{\mathrm{pot}}$$

0.25

Under the assumption that we neglect the Earth's rotation the only contribution to the kinetic energy comes from $v_{escape}$ and neglecting all the other bodies gravitional influence only the potential of the Earth has to be considered meaning

$$\frac{1}{2} m v_{\mathrm{escape}}^2 = \frac{G m m_{\mathrm{Earth}}}{r_{\mathrm{Earth}}}$$

0.25

Solving for $v_{\mathrm{escape}}$:

$$v_{\mathrm{escape}} = \sqrt{\frac{2 G m_{\mathrm{Earth}}}{r_{\mathrm{Earth}}}} = 11.19\,\mathrm{km} \cdot \mathrm{s}^{-1}.$$

0.5

**ii. We assume now that we are in an orbit around the Sun. Calculate the (instantaneous) change in velocity needed to change into the transfer orbit.**
***Hint: use the conservation of energy and angular momentum.*** ***3.5***

*We want to calculate the velocity $v_1$ needed at the closest point to the Sun $R_{Earth}$ to reach Jupiter's orbit at $R_{Jupiter}$. Energy and angular momentum are conserved at every point on the trajectory.*

The total energy consists of kinetic and potential energy and energy conservation reads as

$$E = \frac{m_{\mathrm{rocket}} v_1^2}{2} - \frac{G M_{\mathrm{Sun}} m_{\mathrm{rocket}}}{R_{\mathrm{Earth}}} = \frac{m_{\mathrm{rocket}} v_2^2}{2} - \frac{G M_{\mathrm{Sun}} m_{\mathrm{rocket}}}{R_{\mathrm{Jupiter}}},$$

where $v_2$ is the velocity at the point farthest away. 1

To calculate the angular momentum, we use as a reference point the position of the Sun and see that the position vectors $\vec{R}_{\mathrm{Earth}}$ and $\vec{r}_{\mathrm{Jupiter}}$ are perpendicular to the instantaneous velocities at these points. Therefore the angular momentum conservation reads as

$$L = m_{\mathrm{rocket}} v_1 R_{\mathrm{Earth}} = m_{\mathrm{rocket}} v_2 R_{\mathrm{Jupiter}}.$$

1

We can solve for $v_1$:

$$v_1 = \sqrt{G M_{\mathrm{Sun}} \frac{2 R_{\mathrm{Jupiter}}}{R_{\mathrm{Earth}} \left(R_{\mathrm{Jupiter}} + R_{\mathrm{Earth}}\right)}}.$$

0.5

Since we were on an orbit around the Sun the initial velocity was (see Ai.)

$$v_{\text{init}} = \sqrt{\frac{GM_{\text{Sun}}}{R_{\text{Earth}}}}.$$

0.5

Therefore, you can calculate the difference

$$\Delta v_1 = \sqrt{\frac{GM_{\text{Sun}}}{R_{\text{Earth}}}} \left( \sqrt{\frac{2R_{\text{Jupiter}}}{R_{\text{Earth}} + R_{\text{Jupiter}}}} - 1 \right),$$

$$\Delta v_1 = 8791\,\text{m} \cdot \text{s}^{-1}.$$

0.5

**iii. Calculate the change in velocity to change to Jupiter's orbit (you may ignore the gravitational pull of Jupiter).** ***1***

Analogous considerations as above lead to

$$\Delta v_2 = \sqrt{\frac{GM_{\text{Sun}}}{R_{\text{Jupiter}}}} \left( 1 - \sqrt{\frac{2R_{\text{Earth}}}{R_{\text{Earth}} + R_{\text{Jupiter}}}} \right) = 5643\,\text{m} \cdot \text{s}^{-1}.$$

(points for reasoning and formulas can be awarded in D ii.) 1

**iv. Using the energy density of rocket fuel ($50\,\text{MJ} \cdot \text{kg}^{-1}$) calculate the amount of fuel needed to accomplish all three manoeuvres. Assume the mass of the rocket (including fuel) to be constant. Is this assumption justified?** ***1.5***

The energy needed for all the three stages is equal to the change in kinetic energy.

$$E_1 = \frac{1}{2} m_{\text{rocket}} v_{\text{escape}}^2$$

$$E_2 = \frac{1}{2} m_{\text{rocket}} \left( v_{\text{init}} + \Delta v_1 \right)^2 - \frac{1}{2} m v_{\text{init}}^2$$

$$E_3 = \frac{1}{2} m_{\text{rocket}} \left( v_{\text{final}} + \Delta v_2 \right)^2 - \frac{1}{2} m v_{\text{final}}^2$$

0.5

To get the mass of the fuel needed we divide the total energy $E_{\text{tot}} = E_1 + E_2 + E_3$ by the energy density $\varepsilon$

$$m_{\text{fuel}} = \frac{E_{\text{tot}}}{\varepsilon},$$

0.5

which amounts to $4600\,\text{t}$. 0.25

This is about eight times the weight of the rocket. We see instantly that the assumption that the weight does not change was invalid. 0.25

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# Physics Olympiad

## Final Round

18 - 19 March 2023

| | | |
|---|---|---|
| **Part 2** | **:** | **1 experiment** |
| Duration | : | 90 minutes |
| Total | : | 24 points |
| Authorized material | : | Calculator without database |
| | | Writing and drawing material |

# Good luck!


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## Natural constants

| | | | | |
|---|---|---|---|---|
| Caesium hyperfine frequency | $\Delta\nu_{\mathrm{Cs}}$ | 9.192 631 770 | $\times 10^{9}$ | $\mathrm{s}^{-1}$ |
| Speed of light in vacuum | $c$ | 2.997 924 58 | $\times 10^{8}$ | $\mathrm{m} \cdot \mathrm{s}^{-1}$ |
| Planck constant | $h$ | 6.626 070 15 | $\times 10^{-34}$ | $\mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-1}$ |
| Elementary charge | $e$ | 1.602 176 634 | $\times 10^{-19}$ | $\mathrm{A} \cdot \mathrm{s}$ |
| Boltzmann constant | $k_{\mathrm{B}}$ | 1.380 649 | $\times 10^{-23}$ | $\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-2}$ |
| Avogadro constant | $N_{\mathrm{A}}$ | 6.022 140 76 | $\times 10^{23}$ | $\mathrm{mol}^{-1}$ |
| Luminous efficacy of radiation | $K_{\mathrm{cd}}$ | 6.83 | $\times 10^{2}$ | $\mathrm{cd} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{3} \cdot \mathrm{sr}$ |
| Magnetic constant | $\mu_0$ | 1.256 637 062 12(19) | $\times 10^{-6}$ | $\mathrm{A}^{-2} \cdot \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$ |
| Electric constant | $\varepsilon_0$ | 8.854 187 812 8(13) | $\times 10^{-12}$ | $\mathrm{A}^{2} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-3} \cdot \mathrm{s}^{4}$ |
| Gas constant | $R$ | 8.314 462 618... | | $\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{mol}^{-1} \cdot \mathrm{s}^{-2}$ |
| Stefan-Boltzmann constant | $\sigma$ | 5.670 374 419... | $\times 10^{-8}$ | $\mathrm{K}^{-4} \cdot \mathrm{kg} \cdot \mathrm{s}^{-3}$ |
| Gravitational constant | $G$ | 6.674 30(15) | $\times 10^{-11}$ | $\mathrm{kg}^{-1} \cdot \mathrm{m}^{3} \cdot \mathrm{s}^{-2}$ |
| Electron mass | $m_{\mathrm{e}}$ | 9.109 383 701 5(28) | $\times 10^{-31}$ | kg |
| Neutron mass | $m_{\mathrm{n}}$ | 1.674 927 498 04(95) | $\times 10^{-27}$ | kg |
| Proton mass | $m_{\mathrm{p}}$ | 1.672 621 923 69(51) | $\times 10^{-27}$ | kg |
| Standard acceleration of gravity | $g_{\mathrm{n}}$ | 9.806 65 | | $\mathrm{m} \cdot \mathrm{s}^{-2}$ |

## Experiments

Duration: 90 minutes
Marks: 24 points

### Experiment 2.1: Determine Radius *(24 points)*

#### Introduction

In this task we want to determine the radius of a cylindrical segment. A big part of the challenge is to come up with the experimental setup. However, if you have no idea how to measure this, there are two hints available. Contact the supervisor for the hints. The first hint is a sketch of the experiment and the second hint is a more detailed description with formula. Note: If you need the hints, you will not get any points for the corresponding sub-tasks (you don't get the 5 points for the first hint corresponding to A.i. and for the second hint you don't get the 6 points corresponding to A.ii.). You can only get the second tip after you have taken the first one.
In addition: Only the materials listed below may be used. It is not permitted to use rulers, set squares, compasses or other length measuring instruments! Subtasks in which an unauthorised tool is used will be awarded zero points. Rulers may only be used for drawing graphs or describing experiments.

#### Material

- Wooden cylinder segment with radius $R$ to be determined, see figure Ma.1.
- Aluminium profile with length $L$ and markings at intervals $L/10 \pm 0.0005L$. The mass of the aluminium profile is $(72.6 \pm 0.1)\,\mathrm{g}$.
- Goniometer (angle measuring device).
- Two nails, $(1.53 \pm 0.03)\,\mathrm{g}$ each.
- Tape
- Scissors
- Wooden block

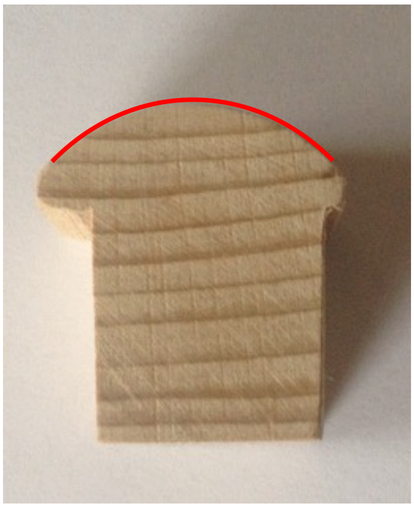

Figure Ma.1: Wooden cylinder segment, the radius is constant along the red line and is to be measured there.

## Tasks

Determine the radius $R$ of the cylinder segment as multiple of the length of the aluminium rod $L$.

### Part A. Think up an experiment *(11 points)*

In this first part, the experimental set-up should be planned, documented and the theory worked out. However, if you have no idea how to measure the radius, there are two hints available. Contact the supervisor for the hints. The first hint is a sketch of the experiment and the second hint is a more detailed description with formula. Note: If you need the hints, you will not get any points for the corresponding sub-tasks (you don't get the 5 points for the first hint corresponding to A.i. and for the second hint you don't get the 6 points corresponding to A.ii.). You can only get the second tip after you have taken the first one.

**i. (5 pts)** Describe your experimental set-up (with sketch) and how you intend to measure the radius $R$.

**ii. (6 pts)** Describe mathematically (with formulae) how you will determine the radius from the measured quantities.

### Part B. Determine Radius *(13 points)*

We now want to determine the radius $R$ as a multiple of $L$ of the cylinder segment.

**i. (5 pts)** Perform the measurement(s) to determine $R$ compared to the length of the aluminium rod $L$.

**ii. (4 pts)** Represent the result graphically.

**iii. (4 pts)** Estimate the measurement accuracy of the radius $R$ with a suitable error analysis.

# Experiments: solutions

## Experiment 2.1: Determine Radius 24

### Introduction

In this task we want to determine the radius of a cylindrical segment. A big part of the challenge is to come up with the experimental setup. However, if you have no idea how to measure this, there are two hints available. Contact the supervisor for the hints. The first hint is a sketch of the experiment and the second hint is a more detailed description with formula. Note: If you need the hints, you will not get any points for the corresponding sub-tasks (you don't get the 5 points for the first hint corresponding to A.i. and for the second hint you don't get the 6 points corresponding to A.ii.). You can only get the second tip after you have taken the first one.
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### Material

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- Scissors
- Wooden block

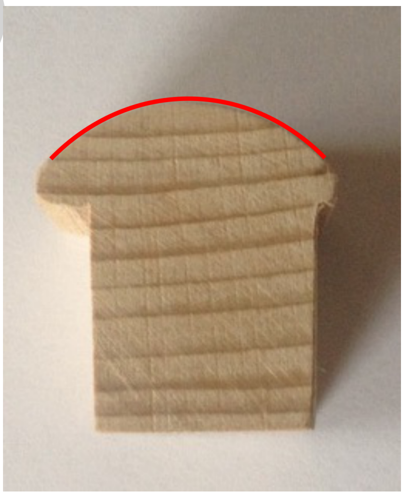

Figure Ma.1: Wooden cylinder segment, the radius is constant along the red line and is to be measured there.

## Tasks

**Determine the radius $R$ of the cylinder segment as multiple of the length of the aluminium rod $L$.**

Part A. Think up an experiment *11*

**In this first part, the experimental set-up should be planned, documented and the theory worked out. However, if you have no idea how to measure the radius, there are two hints available. Contact the supervisor for the hints. The first hint is a sketch of the experiment and the second hint is a more detailed description with formula. Note: If you need the hints, you will not get any points for the corresponding sub-tasks (you don't get the 5 points for the first hint corresponding to A.i. and for the second hint you don't get the 6 points corresponding to A.ii.). You can only get the second tip after you have taken the first one.**

**i. Describe your experimental set-up (with sketch) and how you intend to measure the radius $R$.** ***5***

*The idea is to have a balance with the aluminium rod and have it balancing in equilibrium on the cylinder. Then by adding a bit of weight, the center of mass (CMS) shifts and the rod tilts a bit to the side by rolling by a distance $s$ on the cylinder. Knowing the angle of this tilting $\alpha$ and comparing it with the shift of the CMS, we can conclude on the radius.*
*Note that some explanations or ideas might also appear in the next subtask, then the points are are also given.*

| | |
|---|---|
| Idea of balance (give this points also if not explicitly stated but thought obvious) | 2 |
| Idea of shifting CMS (give this points also if not explicitly stated but thought obvious) | 2 |

Clear drawing

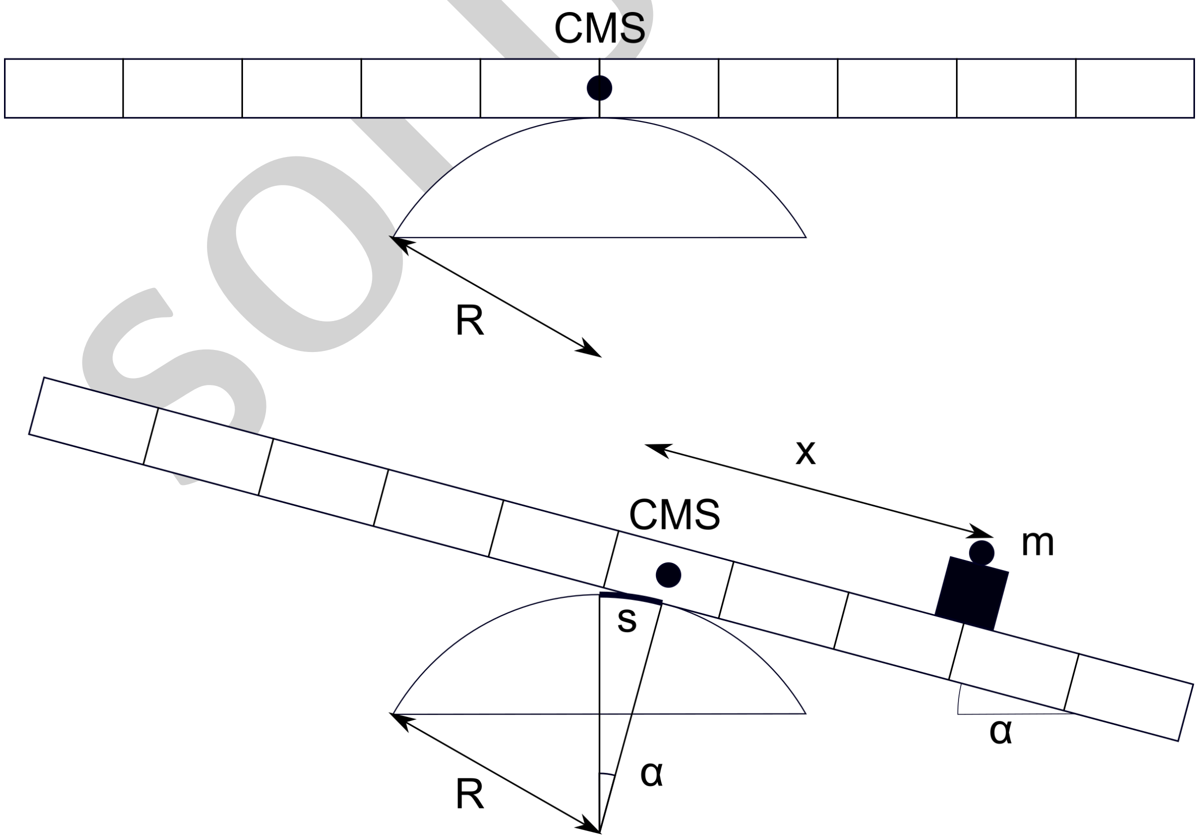

Figure A.1: Idea of how to measure the radius. Explanation see text, variables defined and used in next subtask.

1

***Alternative solution:*** ***(5)***

*A simpler, but potentially less precise approach involves measuring the length of the arc and the corresponding angle and then conclude on the radius.*

*Idea of measuring arc and angle.* (1)

*Clear drawing.* (1)

*Suggestions for increasing precision of measurement.* (1)

*Having a suggestion that does not include the construction of the center of the cylinder (due to the rather small segment provided by the mushroom-like wooden piece, the construction of the center is very imprecise).* (2)

**ii. Describe mathematically (with formulae) how you will determine the radius from the measured quantities.** ***6***

*The idea is to have a balance with the aluminium rod and have it balancing in equilibrium on the cylinder. Then by adding a bit of weight, the center of mass (CMS) shifts and the rod tilts a bit to the side by rolling by a distance $s$ on the cylinder. Knowing the angle of this tilting $\alpha$ and comparing it with the shift of the CMS, we can conclude on the radius. The overall function is (explanation see at point distribution, however, the final formula is not worth any points as such):*

$$R = \frac{s}{\alpha} = \frac{xm}{(M+m)\alpha} \tag{A.2}$$

*Note that some explanations, ideas might also appear in the previous subtask, then the points are also given.*

CMS without additional mass is in middle of rod (give this points also if not explicitly stated but thought obvious) 1

When adding the small mass $m$ at a distance $x$ from the middle of the rod, the CMS moves by a distance $s = \frac{xm}{M+m}$ where $M$ is the mass of the rod. If the approximation $M \ll m$ is made and correspondingly $s = \frac{xm}{M}$, only 1 point is given. 2

Measuring angle $\alpha$ 1

This distance $s$ is connected with the tilting angle $\alpha$ as $s = R\alpha$ 1

Technical detail: either it is explicitly stated that the rod without additional mass $m$ is set such that it is very well balanced or an initial tilt $\alpha_0$ is measured and $\alpha$ is referenced to that value $\alpha_0$. This point is also given if it is staten in one of the next tasks. 1

***Alternative solution:*** ***(6)***

*Continuing on the alternative approach measuring the arc and the corresponding angle (note, there are different possibilities):*

| | |
|---|---|
| *Arc $s$ and angle $\alpha$ are connected as $s = r\alpha$* | (1) |
| *Mention measure arc $s$ and angle $\alpha$ (also give these points if obvious otherwise)* | (1) |
| *Explanations on careful measurement* | (1) |
| *Explanations on how to do the measurements without constructing the center of the arc. For example by not measuring the angle of the segment (corresponding to the arc) but measuring the angle of a side of the wooden cylinder with respect to some reference (e.g. the table).* | (3) |

## Part B. Determine Radius *13*

**We now want to determine the radius $R$ as a multiple of $L$ of the cylinder segment.**

**i. Perform the measurement(s) to determine $R$ compared to the length of the aluminium rod $L$.** ***5***

*The measurement as described above is performed now (for all the part B, the measurements are independent of the measurement method and the points distributed accordingly). If some points from the previous task get clarified, feel free to include it in the marking above.*
*The measurement should roughly look like (including an offset when there is no nail on the balance of 0.5°)*

| $s/L$ | $\alpha/°$ | *Nr nails* | $R/L$ | $\sigma_R$ |
|---|---|---|---|---|
| $-0.1$ | $-2$ | 1 | 0.059 | 0.019 |
| $-0.2$ | $-4.5$ | 1 | 0.053 | 0.008 |
| $-0.3$ | $-6.5$ | 1 | 0.055 | 0.006 |
| $-0.4$ | $-8$ | 1 | 0.059 | 0.005 |
| 0.1 | 1.5 | 1 | 0.079 | 0.031 |
| 0.2 | 3 | 1 | 0.079 | 0.015 |
| 0.3 | 5.5 | 1 | 0.065 | 0.007 |
| 0.4 | 7.5 | 1 | 0.063 | 0.005 |
| 0.1 | 3.5 | 2 | 0.066 | 0.016 |
| $-0.1$ | $-4.5$ | 2 | 0.051 | 0.012 |
| 0.2 | 7 | 2 | 0.055 | 0.006 |
| $-0.2$ | $-8.5$ | 2 | 0.066 | 0.008 |

*Note that 2 nails can only be placed at the $s = 0.1L$ and $s = 0.2L$ The obtained mean is 0.062.*
*For later, the expected error according to Gaussian error propagation is also included in this table.*

| | |
|---|---|
| Number of measurements/datapoints: 1 measurement: 0 pt, 2 or 3 measurements: 1 pt, 4 or more measurements: 2 pt | 2 |
| The exact value is $0.071L$. Give points if obtained value between 0.057 and 0.086 ($\pm 20\%$) | 1 |
| Value obtained by some kind of averaging (either averaging of individual values or trend line in graph). There are mainly two ways how to change the experimental parameters: The easier is to move the mass $m$ on the aluminium rod. The other method is to vary the weight $m$. Both are considered ok, nevertheless the second one is a bit trickier for later linearization. Note: Zero points if single measurement. | 1 |
| Values taken on both sides of the balance (i.e. at positive and negative $s$ and correspondingly positive and negative $\alpha$). This makes the evaluation more precise. | 1 |

**ii. Represent the result graphically.** ***4***

*Graphical data representation. It is unimportant whether $R$ for different measurements is plotted or a graph which connects $\alpha$ and $s$ (most conclusive) or something else. Here the blue dots are the measurements with only one nail, which are also fitted. The orange dots are with two nails.*

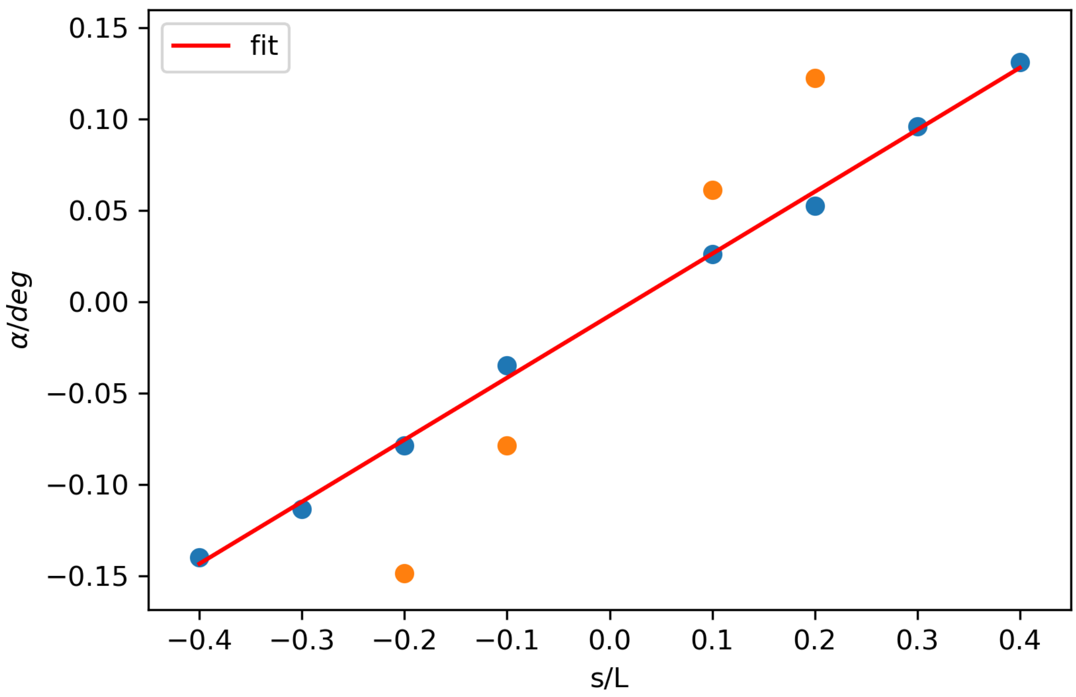


Figure B.1: visualized data.

| | |
|---|---|
| Data correctly plotted | 1 |
| Graphic representation: axis labelled | 0.5 |
| Units and scale clear | 0.5 |
| Data points obvious (not connected with line) | 1 |
| Big graph | 0.5 |
| Axis drawn by ruler | 0.5 |

**iii. Estimate the measurement accuracy of the radius $R$ with a suitable error analysis.** *4*

*The actual estimation of the measuring error and also the method is not so important. However, it should be clear how the value is obtained and what enters the calculation.*
*If the error estimation is done graphically in a linearized data evaluation by roughly estimating the spread of the data: 3 pt (full points for the method, if clear how it was done). As mentioned earlier, there are basically two methods how to do a linarized data evaluation: either move the position of the additional weight $m$ or change the weight, both are ok, for the equation, see* (A.2).
*The following scheme includes the point distribution for computing the standard deviation from computing $R$ several times. For the method of Gaussian error propagation, see the alternative solution below.*

| | |
|---|---|
| Computing the standard deviation of the sample $\sigma = \sqrt{\sum_i \left(R_i - \bar{R}\right)^2}$. | 1 |

| | |
|---|---|
| Note, this point can only be obtained if several datapoints are included in the error analysis. Dividing the std by the square of the number of points | 1 |
| Somehehow state that for small $\alpha$ the error is big and hence they dominate the estimated error. If anyway only big $\alpha$ are measured ($\lvert\alpha\rvert > 5°$), this is not an issue and this point is given. | 1 |
| Obtained or guessed accuracy of about maximal 0.005 (if larger, the measurement is considered not to be done careful enough). From the measurement here, a value of 0.003 is obtained (including all datapoints). | 1 |
| ***Alternative solution:*** | *(4)* |
| *Reasonable error propagation (sum of absolute values or rms) for a single point.* | (1) |
| *Note, this point can only be obtained if several datapoints are included in the error analysis. Error propagation for the average of the data points.* | (1) |
| *Error of $\alpha$ of $\pm 0.5°$ considered. For the other variables ($m$, $M$, $x$) it is not necessary to include them as their error is negligible compared to error of $\alpha$ (however, also ok if taken into account).* | (1) |
| *Obtained or guessed accuracy of about maximal 0.005 (if larger, the measurement is considered not to be done careful enough).* | (1) |

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# Physics Olympiad

## Final Round

18 - 19 March 2023

| | | |
|---:|---|---|
| **Part 3** | **:** | **1 experiment** |
| Duration | : | 90 minutes |
| Total | : | 24 points |
| Authorized material | : | Calculator without database |
| | | Writing and drawing material |

# Good luck!


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Staatssekretariat für Bildung, Forschung und Innovation
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Swiss Academy of Sciences
Swiss Physical Society
Università della Svizzera italiana
Universität Bern FB Physik/Astronomie
Universität Zürich FB Physik Mathematik

## Natural constants

| | | | | |
|---|---|---|---|---|
| Caesium hyperfine frequency | $\Delta\nu_{\mathrm{Cs}}$ | 9.192 631 770 | $\times 10^{9}$ | $\mathrm{s}^{-1}$ |
| Speed of light in vacuum | $c$ | 2.997 924 58 | $\times 10^{8}$ | $\mathrm{m} \cdot \mathrm{s}^{-1}$ |
| Planck constant | $h$ | 6.626 070 15 | $\times 10^{-34}$ | $\mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-1}$ |
| Elementary charge | $e$ | 1.602 176 634 | $\times 10^{-19}$ | $\mathrm{A} \cdot \mathrm{s}$ |
| Boltzmann constant | $k_{\mathrm{B}}$ | 1.380 649 | $\times 10^{-23}$ | $\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{s}^{-2}$ |
| Avogadro constant | $N_{\mathrm{A}}$ | 6.022 140 76 | $\times 10^{23}$ | $\mathrm{mol}^{-1}$ |
| Luminous efficacy of radiation | $K_{\mathrm{cd}}$ | 6.83 | $\times 10^{2}$ | $\mathrm{cd} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-2} \cdot \mathrm{s}^{3} \cdot \mathrm{sr}$ |
| Magnetic constant | $\mu_0$ | 1.256 637 062 12(19) | $\times 10^{-6}$ | $\mathrm{A}^{-2} \cdot \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$ |
| Electric constant | $\varepsilon_0$ | 8.854 187 812 8(13) | $\times 10^{-12}$ | $\mathrm{A}^{2} \cdot \mathrm{kg}^{-1} \cdot \mathrm{m}^{-3} \cdot \mathrm{s}^{4}$ |
| Gas constant | $R$ | 8.314 462 618... | | $\mathrm{K}^{-1} \cdot \mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{mol}^{-1} \cdot \mathrm{s}^{-2}$ |
| Stefan-Boltzmann constant | $\sigma$ | 5.670 374 419... | $\times 10^{-8}$ | $\mathrm{K}^{-4} \cdot \mathrm{kg} \cdot \mathrm{s}^{-3}$ |
| Gravitational constant | $G$ | 6.674 30(15) | $\times 10^{-11}$ | $\mathrm{kg}^{-1} \cdot \mathrm{m}^{3} \cdot \mathrm{s}^{-2}$ |
| Electron mass | $m_{\mathrm{e}}$ | 9.109 383 701 5(28) | $\times 10^{-31}$ | kg |
| Neutron mass | $m_{\mathrm{n}}$ | 1.674 927 498 04(95) | $\times 10^{-27}$ | kg |
| Proton mass | $m_{\mathrm{p}}$ | 1.672 621 923 69(51) | $\times 10^{-27}$ | kg |
| Standard acceleration of gravity | $g_{\mathrm{n}}$ | 9.806 65 | | $\mathrm{m} \cdot \mathrm{s}^{-2}$ |

# Experiments

Duration: 90 minutes
Marks: 24 points

## Experiment 3.1: The real Transformer *(24 points)*

### Introduction

In this task we want to investigate the behaviour of a transformer when it is loaded and has to supply power. A transformer consists of two coils which are coupled together by a common iron core (see figure In.2). If an alternating voltage is applied to one of the coils (primary coil), the alternating voltage generates a magnetic field in the iron core that changes over time, which in turn generates a voltage in the other coil (secondary coil). The formula for the ideal transformer states that between the number of windigns $N_1$ and $N_2$ of the primary and secondary coils as well as their voltages $U_1$ and $U_2$ the relationship

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = \text{const.}$$

applies. From the conservation of energy it then follows for the currents that

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \text{const.}$$

For some designs as well as for small currents, these formulae still apply with good approximation. The validity of the formula depends on whether the coils are strongly coupled magnetically, i.e. whether the magnetic field generated by the primary coil still flows through the iron core and therefore also through the secondary coil, or seeks a "new" path outside the iron core. These "new" paths are called stray fluxes.
Here we want to investigate the case where the stray fluxes are no longer negligible and the formula above no longer applies.
In order to understand and describe the transformer more precisely, we have to take a closer look at the coils. According to the law of induction, the voltage $U$ of a coil is proportional to the time change of the magnetic flux $\Phi$ (where $\Phi = BA$ with $B$ the magnetic field strength, $A$ the cross-sectional area and $N$ the number of windings). If we consider a fixed frequency $\omega$ of the alternating current and only look at the peak or effective values, the formula simplifies to

$$U = \omega N \Phi.$$

Since magnetic fields do not have a beginning and an end but are closed field lines, the magnetic flux $\Phi_1$ generated by the primary coil must somehow flow back outside the primary coil. Part of it flows through the iron core and passes through the secondary coil $\Phi_2$, but depending on the load on the secondary coil (i.e. resistance $R$ at the secondary coil), a considerable part also flows as leakage flux $\Phi_S$ outside the core.

$$\begin{aligned} \Phi_1 &= \Phi_2 + \Phi_S \\ \frac{U_1}{N_1} &= \frac{U_2}{N_2} + \Phi_S \omega \end{aligned} \tag{In.1}$$

A description of the construction is expected for all tasks. Attention: the current through the coils must NEVER exceed 0.5 A, this can be achieved by keeping the primary voltage always below 2 V. For safety's sake, always use the fuses.
Another hint: Read through all the tasks first.

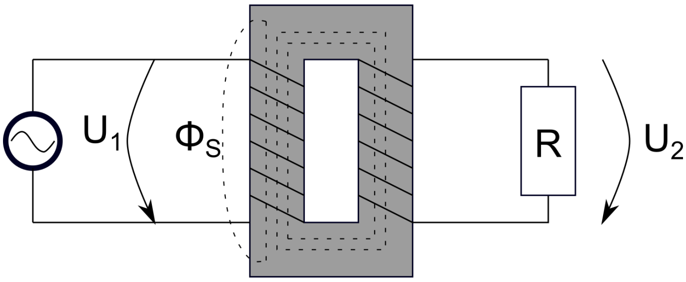


Figure In.2: Transformer with primary coil (left, connected to AC source) and secondary coil (right, with resistor $R$). The dashed lines represent the magnetic field. Part of the field does not pass through the iron core (left of the primary coil), this is the leakage flux.

## Material

- 2 multimeters
- A set of resistors ($1\,\Omega$, $15\,\Omega$, $22\,\Omega$, $68\,\Omega$, $1000\,\Omega$)
- 1 alternating current source
- 1 transformer with two identical coils (please use only the two outer sockets)
- 1 additional coil
- 1 fuse 0.5 A (if this blows, please ask the supervisor for a replacement)
- 10 crocodile clips (5 red and 5 black)
- 12 lab cables (6 red and 6 black)
- Tape
- Scissors

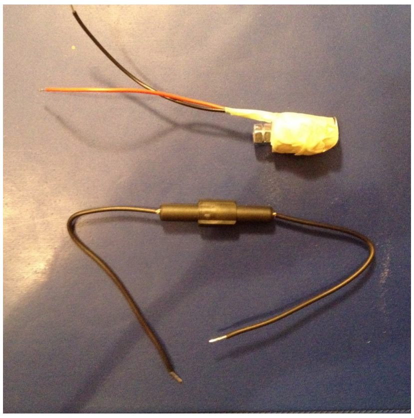

Figure Ma.3: Top: additional coil, bottom: fuse

## Tasks

A description of the construction is expected for all tasks. Attention: the current through the coils must NEVER exceed 0.5 A, this can be achieved by keeping the primary voltage always below 2 V. For safety, please always use the fuses.

## Part A. Apply Load to the transformer *(7.5 points)*

First we want to get to know the behaviour of the transformer.

**i. (4.5 pts)** Measure the primary voltage $U_1$ and secondary voltage $U_2$ for different loads (ohmic resistance $R$ at the secondary circuit) of the transformer. Calculate or measure the current $I_2$ in the secondary coil. Perform at least 9 measurements.

**ii. (3 pts)** Make a graph with the ratio $U_2/U_1$ as a function of $I_2$.

## Part B. Ratio of voltages *(3 points)*

Now we want to show that the ratio $U_2/U_1$ is independent of the primary voltage for a given load.

**i. (1 pt)** At what load (ohmic resistance $R$ on the secondary circuit) you used is the secondary voltage approximately half the primary voltage?

**ii. (2 pts)** Show with a suitable measurement that with that resistance, $2U_2 = U_1$ is largely independent of the primary voltage. Please ensure to stay below 2 V as primary voltage.

## Part C. Stray flux *(13.5 points)*

Now let us show that the leakage flux is indeed responsible for the decrease of the secondary voltage at higher load. You cannot measure the whole leakage flux as such, but you can measure a quantity which is proportional to the leakage flux. We will refer to this measurable quantity as $U_\mathrm{S}$ in the following.
Accordingly, we cannot show exactly that the leakage fluxes are responsible for the decrease of the secondary voltage at higher loads, but only that a reduction of the secondary voltage causes a proportionally large increase of the leakage flux. Since the magnetic fluxes cannot be measured directly but only quantities proportional to them (in our experiment induced voltages in different coils), equation (In.1) can be rewritten as follows:

$$\begin{aligned}\Phi_1 &= \Phi_2 + \Phi_\mathrm{S} \\ \frac{U_1}{C_1} &= \frac{U_2}{C_2} + \frac{U_\mathrm{S}}{C_\mathrm{S}}.\end{aligned} \tag{C.1}$$

For this sub-task you can use your measurements from previous tasks. However, it must be clear which data originate from earlier data and which are newly measured!

**i. (5 pts)** Find a way to measure the stray flux (not calculating it from $U_1$ and $U_2$) and describe your idea.

**ii. (4.5 pts)** Perform the correpsonding measurement and show with a suitable graph that $U_\mathrm{S}/U_1$ depends linearly on $U_2/U_1$ (This won't be exactly the case as there are additional effects).

**iii. (4 pts)** Calculate from your measurement $C_1/C_2$ and interpret the value you found.

## Appendix 1: The Voltage Source

For the experiment we need an alternating current (AC) source, which is briefly explained below, see figure Ax1.1. Note the following important points:

- ONLY the AC voltage may be used, the DC voltage would destroy the transformer immediately!
- To keep the current below 0.5 A, the voltage should always be below 2 V!

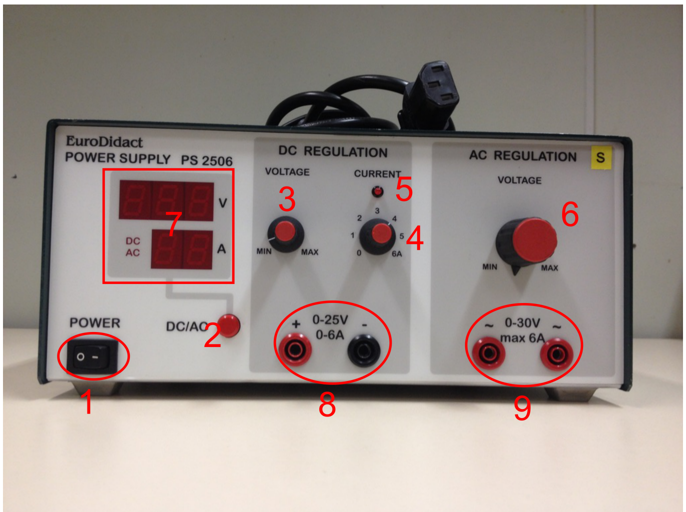


Figure Ax1.1: Explanations: 1: On-off switch, 2: Switch display (7) to AC (alternating voltage) or DC (direct voltage), always have AC, 3: Voltage regulation DC, 4: Current limitation DC (not for AC voltage!), 5: Overload for DC (not for AC voltage), 6: Voltage regulation AC, 7. display for current and voltage, 8: DC outputs, 9: AC outputs.

## Appendix 2: The Multimeter

For the measurement of voltage, current and resistance we use a multimeter. Two models are used, see figures Ax2.1 and Ax2.2. Although there are differences in operation, there are a few points that need to be considered (also to protect the multimeter).

- One cable must always be in the COM socket. This is the zero point (usually the black cable).
- The input for voltage measurement (and sometimes small currents) has high impedance, i.e. only a small current flows through the multimeter.
- The input for current measurement has low impedance, i.e. a large current can flow through the multimeter.
- In general, when measuring currents: the current flows through the multimeter, which has a much lower resistance than when measuring a voltage. This means that large currents can flow and destroy the multimeter (or the voltage source)! Therefore, the multimeter in this configuration must NEVER be connected in parallel to the voltage source, but only in series with a load!

Figure Ax2.1: With this multimeter, the measuring range must be set manually, e.g. a different voltage range must be selected depending on the voltage. Explanations: 1: on-off switch, 2: COM socket, where the black cable belongs, 3: socket for the second cable for voltage or resistance measurement, 4: socket for the second cable for current measurement up to MAXIMUM 0.2 A, 5: socket for the second cable for current measurement up to MAXIMUM 20 A, 6: voltage measurement DC voltage, different measuring ranges, 7: voltage measurement AC voltage, different measuring ranges, 8: current measurement AC current (attention: for 0.2 A use other socket), different measuring ranges, 9: current measurement DC (attention: for 0.2 A use other socket), different measuring ranges, 10: resistance measurement, different measuring ranges, 11: display.

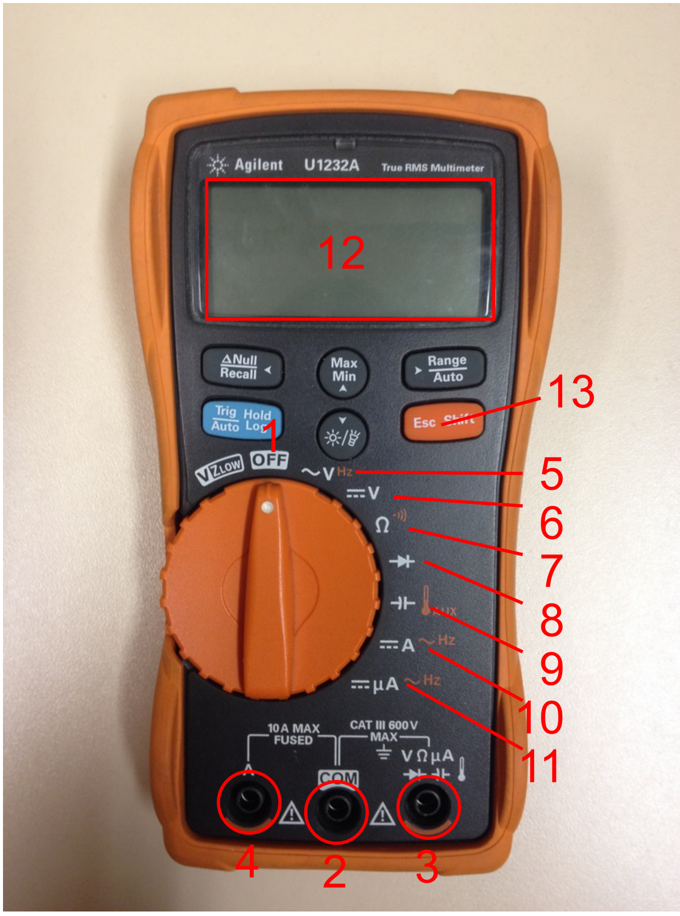


Figure Ax2.2: With this multimeter, the measuring range is set automatically. Explanations: 1: multimeter switched off, 2: COM socket, where the black cable belongs, 3: socket for the second cable for voltage or resistance measurement and current measurement up to MAXIMUM 0.6 mA, 4: socket for the second cable for current measurement up to MAXIMUM 10 A, 5: AC voltage measurement, 6: DC voltage measurement, 7: resistance measurement, 8, 9: not relevant here, 10: heavy current measurement, up to 10 A. To switch from direct current to alternating current, press the orange button (13), 11: small current measurement, up to 0.6 mA. To switch from direct current to alternating current, press the orange button (13), 12: display, 13: change certain measurements (orange signs on the wheel).

# Experiments: solutions

## Experiment 3.1: The real Transformer 24

### Introduction

In this task we want to investigate the behaviour of a transformer when it is loaded and has to supply power.
A transformer consists of two coils which are coupled together by a common iron core (see figure In.2). If an alternating voltage is applied to one of the coils (primary coil), the alternating voltage generates a magnetic field in the iron core that changes over time, which in turn generates a voltage in the other coil (secondary coil). The formula for the ideal transformer states that between the number of windigns $N_1$ and $N_2$ of the primary and secondary coils as well as their voltages $U_1$ and $U_2$ the relationship

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applies. From the conservation of energy it then follows for the currents that

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For some designs as well as for small currents, these formulae still apply with good approximation. The validity of the formula depends on whether the coils are strongly coupled magnetically, i.e. whether the magnetic field generated by the primary coil still flows through the iron core and therefore also through the secondary coil, or seeks a ”new” path outside the iron core. These ”new” paths are called stray fluxes. Here we want to investigate the case where the stray fluxes are no longer negligible and the formula above no longer applies.
In order to understand and describe the transformer more precisely, we have to take a closer look at the coils. According to the law of induction, the voltage $U$ of a coil is proportional to the time change of the magnetic flux $\Phi$ (where $\Phi = BA$ with $B$ the magnetic field strength, $A$ the cross-sectional area and $N$ the number of windings). If we consider a fixed frequency $\omega$ of the alternating current and only look at the peak or effective values, the formula simplifies to

$$U = \omega N \Phi.$$

Since magnetic fields do not have a beginning and an end but are closed field lines, the magnetic flux $\Phi_1$ generated by the primary coil must somehow flow back outside the primary coil. Part of it flows through the iron core and passes through the secondary coil $\Phi_2$, but depending on the load on the secondary coil (i.e. resistance $R$ at the secondary coil), a considerable part also flows as leakage flux $\Phi_S$ outside the core.

$$\begin{aligned} \Phi_1 &= \Phi_2 + \Phi_S \\ \frac{U_1}{N_1} &= \frac{U_2}{N_2} + \Phi_S \omega \end{aligned} \tag{In.1}$$

A description of the construction is expected for all tasks. Attention: the current through the coils must NEVER exceed 0.5 A, this can be achieved by keeping the primary voltage always below 2 V. For safety's sake, always use the fuses.
Another hint: Read through all the tasks first.

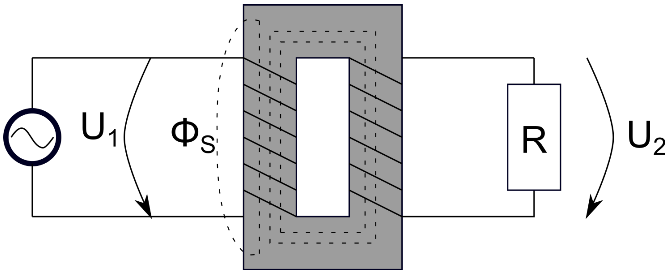


Figure In.2: Transformer with primary coil (left, connected to AC source) and secondary coil (right, with resistor $R$). The dashed lines represent the magnetic field. Part of the field does not pass through the iron core (left of the primary coil), this is the leakage flux.

## Material

- 2 multimeters
- A set of resistors ($1\,\Omega$, $15\,\Omega$, $22\,\Omega$, $68\,\Omega$, $1000\,\Omega$)
- 1 alternating current source
- 1 transformer with two identical coils (please use only the two outer sockets)
- 1 additional coil
- 1 fuse 0.5 A (if this blows, please ask the supervisor for a replacement)
- 10 crocodile clips (5 red and 5 black)
- 12 lab cables (6 red and 6 black)
- Tape
- Scissors

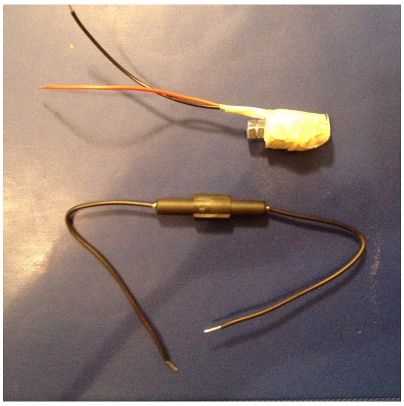

Figure Ma.3: Top: additional coil, bottom: fuse

## Tasks

**A description of the construction is expected for all tasks. Attention: the current through the coils must NEVER exceed $0.5\,\mathrm{A}$, this can be achieved by keeping the primary voltage always below $2\,\mathrm{V}$. For safety, please always use the fuses.**

### Part A. Apply Load to the transformer — *7.5*

**First we want to get to know the behaviour of the transformer.**

**i. Measure the primary voltage $U_1$ and secondary voltage $U_2$ for different loads (ohmic resistance $R$ at the secondary circuit) of the transformer. Calculate or measure the current $I_2$ in the secondary coil. Perform at least 9 measurements.** *4.5*

*The idea is to connect the secondary coil with different resistors and measure $U_1$ and $U_2$. Note that one has to combine different resistors to get enough measurement points. The collected data might look like (other resistors are used than the given in the exam, however, the actual value of the resistor is unimprtant (more precisely, the ohmic resistor of the secondary coil would also count), only $U_2/U_1$ is relevant):*

| $R/\Omega$ | *Combination* | $U_1/\mathrm{V}$ | $U_2/\mathrm{V}$ | $I_2/\mathrm{mA}$ | $U_S/\mathrm{mV}$ | $U_2/U_1$ | $U_S/U_1$ |
|---|---|---|---|---|---|---|---|
| 0 | $0\,\Omega$ | 1.75 | 0.00 | 45.4 | 289 | 0.000 | 145 |
| 1 | $1\,\Omega$ | 1.75 | 0.05 | 45.0 | 287 | 0.0290 | 144 |
| 8.9 | $15\,\Omega + 22\,\Omega$ *parallel* | 1.74 | 0.38 | 40.1 | 259 | 0.218 | 129 |
| 12.3 | $15\,\Omega + 68\,\Omega$ *parallel* | 1.74 | 0.49 | 38.3 | 246 | 0.282 | 121 |
| 15 | $15\,\Omega$ | 1.74 | 0.57 | 36.7 | 237 | 0.328 | 116 |
| 22 | $22\,\Omega$ | 1.75 | 0.77 | 33.0 | 212 | 0.440 | 101 |
| 37 | $15\,\Omega + 22\,\Omega$ | 1.76 | 1.04 | 26.7 | 172 | 0.581 | 77.8 |
| 68 | $68\,\Omega$ | 1.79 | 1.31 | 18.7 | 122 | 0.732 | 48.6 |
| 83 | $15\,\Omega + 68\,\Omega$ | 1.79 | 1.38 | 16.1 | 107 | 0.771 | 40.23 |
| 90 | $22\,\Omega + 68\,\Omega$ | 1.80 | 1.41 | 15.2 | 101 | 0.783 | 36.7 |
| 105 | $15\,\Omega + 22\,\Omega + 68\,\Omega$ | 1.81 | 1.46 | 13.4 | 91 | 0.807 | 30.9 |
| 1000 | $1000\,\Omega$ | 1.86 | 1.70 | 1.54 | 38 | 0.914 | 1.61 |
| $\infty$ | *open circuit* | 1.86 | 1.72 | 0.00 | 35 | 0.925 | 0 |

*Note that some of the data above will be relevant later, but is collected here for completeness.*

Clear description of the experiment (The idea is to connect the secondary coil with different resistors and measure $U_1$ and $U_2$. ) 1

Number of measurements, if less than 9, subtract 0.5 P for each missing point. 1

Quantities correctly measured, i.e. the voltages over the corresponding coils. The current has to be measured or if computed with the resistors, the resistors have to be measured (as they have an uncertinty of $\pm 5\%$) 1

| | |
|---|---|
| Open circuit included | 0.25 |
| 1000 Ohm included | 0.25 |
| Shortcut included or argued why this is not useful to measure (i.e. because the resistance of the transformer is non negligible for such a small resistance) | 0.25 |
| 1 Ohm included or argued why this is not useful to measure (i.e. because the resistance of the transformer is non negligible for such a small resistance) | 0.25 |
| Three resistances between 5 and 30 Ohm included | 0.25 |
| Two resistances between 50 and 120 Ohm included | 0.25 |

**ii. Make a graph with the ratio $U_2/U_1$ as a function of $I_2$.** *3*

*Here the data obtained from the previous task is represented.*

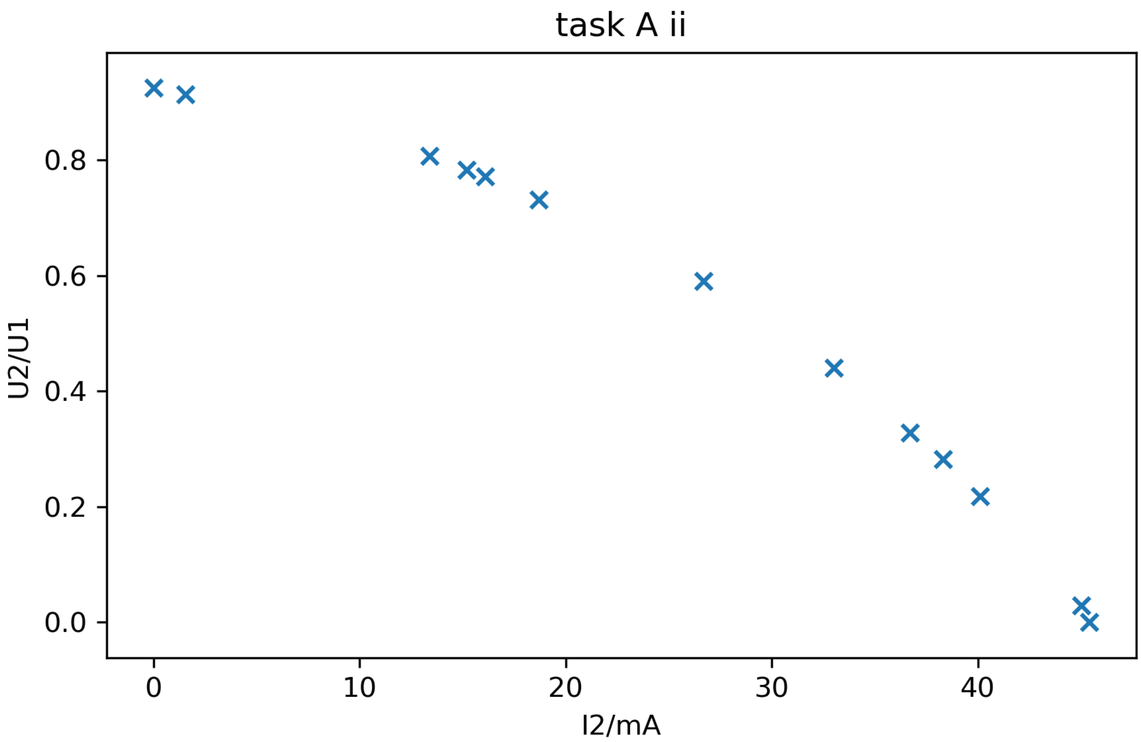


Figure A.1: Expected graph.

| | |
|---|---|
| Data points correctly entered (for the 1 Ohm resistor and the short cut there might be weird results if the current is computed as $U_2/R$. We do not care about this.) | 0.5 |
| Graphic representation: axis labelled | 0.5 |
| Units and scale clear | 0.5 |

| | |
|---|---|
| Data points obvious (not connected with line) | 0.5 |
| Big graph | 0.5 |
| Axis drawn by ruler | 0.5 |

## Part B. Ratio of voltages — *3*

**Now we want to show that the ratio $U_2/U_1$ is independent of the primary voltage for a given load.**

**i. At what load (ohmic resistance $R$ on the secondary circuit) you used is the secondary voltage approximately half the primary voltage?** *1*

*The idea is to find a resistor (or a combination of several), where $2U_2 = U_1$*

| | |
|---|---|
| Find correct resistance, it should be either 22 Ohm or 37 (= 15+22) Ohm. Give this point also, if the students did not use these two resistances but quote their closest resistance. | 1 |

**ii. Show with a suitable measurement that with that resistance, $2U_2 = U_1$ is largely independent of the primary voltage. Please ensure to stay below 2 V as primary voltage.** *2*

*Knowing the right resistor, we change $U_1$ and check whether the ratio $U_1/U_2$ stays the same. Note, that the points in this sub tasks are distributed independent whether the resistor is correct or not.*

| | |
|---|---|
| Quick description of experiment (it is enough to state that it is the same as above but fixed $R$ and variable $U_1$) | 0.5 |
| Number of measurements: at least 3 | 0.5 |
| Range of $U_1$: lower than 0.9 V and higher than 1.6 V | 0.5 |
| Document/Indicate that the ratio $U_2/U_1$ stays almost the same (needs to be indicated explicitly) | 0.5 |

## Part C. Stray flux — *13.5*

**Now let us show that the leakage flux is indeed responsible for the decrease of the secondary voltage at higher load. You cannot measure the whole leakage flux as such, but you can measure a quantity which is proportional to the leakage flux. We will refer to this measurable quantity as $U_S$ in the following.**
**Accordingly, we cannot show exactly that the leakage fluxes are responsible for the decrease of the secondary voltage at higher loads, but only that a reduction of the secondary voltage causes a proportionally large increase of the leakage flux. Since the magnetic fluxes cannot be measured directly but only quantities proportional to them (in our experiment induced voltages in different coils), equation (In.1) can be rewritten as follows:**

$$\begin{aligned} \Phi_1 &= \Phi_2 + \Phi_S \\ \frac{U_1}{C_1} &= \frac{U_2}{C_2} + \frac{U_S}{C_S}. \end{aligned} \tag{C.1}$$

**For this sub-task you can use your measurements from previous tasks. However, it must be clear which data originate from earlier data and which are newly measured!**

**i. Find a way to measure the stray flux (not calculating it from $U_1$ and $U_2$) and describe your idea.** ***5***

*Using the additional small coil, one can measure the induced voltage. That voltage is proportional to the flux $\Phi_C$ going though the coil. This flux is proportional to the entire flux.*

Idea of using the small coil (best aside of the primary coil). Needs to be clear from description. 2

The voltage $U_\mathrm{S}$ measured at this coil is proportional to the flux though the coil. Needs to be clear from description. 1

The flux though the coil is proportional to the entire flux not going though the iron. Needs to be clear from description. 2

**ii. Perform the correpsonding measurement and show with a suitable graph that $U_\mathrm{S}/U_1$ depends linearly on $U_2/U_1$ (This won't be exactly the case as there are additional effects).** ***4.5***

*We rearrange (equation* (C.1)*)*

$$\frac{U_S}{U_1} = \frac{C_S}{C_1} - \frac{C_S}{C_2}\frac{U_2}{U_1},$$

*which is a linear equation $y = \frac{U_S}{U_1}$ and $x = \frac{U_2}{U_1}$. If the plot $x$ vs. $y$ is linear, equation* (In.1) *is fulfilled. Note that one shoul subtract some offset voltage from $U_S$ (i.e. the measured voltage at the small coil when the circuit is open). Nevertheless, we do not care whether this is done or not. The plot should look as follows:*

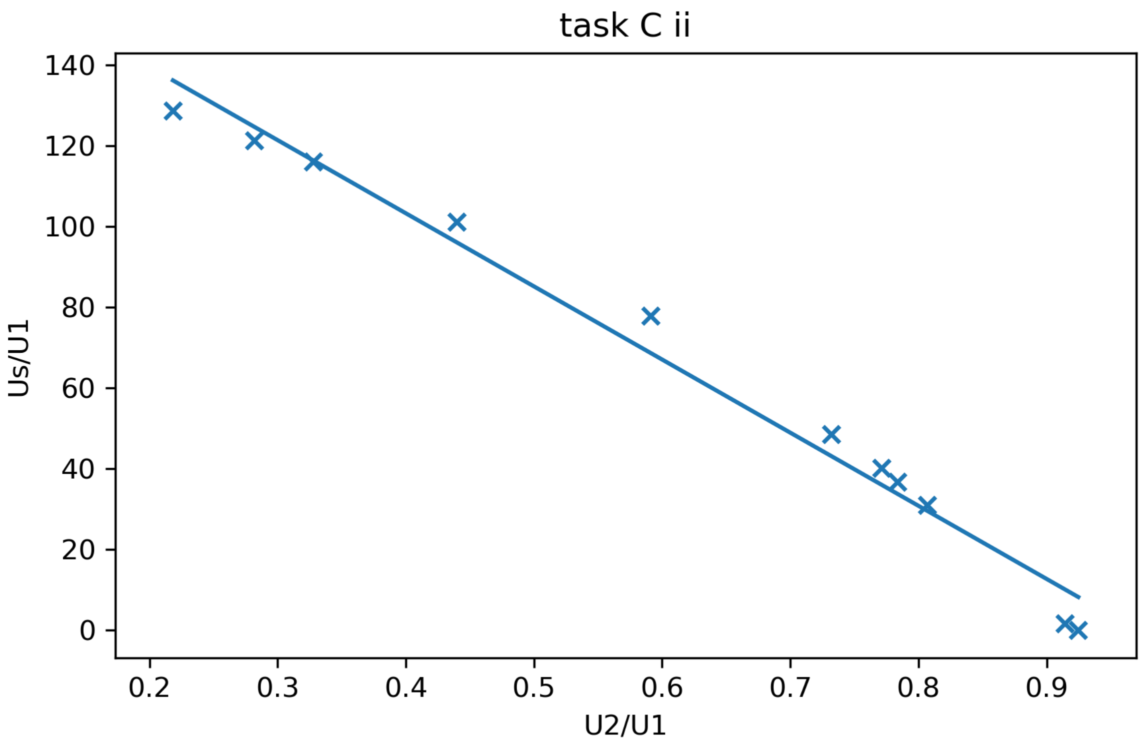


Figure C.2: Expected graph.

*Points are distributed as:*

| | |
|---|---|
| Finding the linearized equation (or something similar) | 1 |
| Concluding that a linear plot shows the expected behaviour (need to be stated explicitly) | 1 |
| Measuring $U_S$ for at least 4 values (together with $U_1$ and $U_2$ or using $U_1$ and $U_2$ from previous tasks) | 1 |
| Graphic representation: axis labelled | 0.25 |
| Units and scale clear | 0.25 |
| Data points obvious (not connected with line) | 0.5 |
| Big graph | 0.25 |
| Axis drawn by ruler | 0.25 |
| **iii. Calculate from your measurement $C_1/C_2$ and interpret the value you found.** | *4* |
| *From the linear dependence between $U_S/U_1$ as function of $U_2/U_1$ we find that $C_1/C_2$ is equal to minus the y-axis intersection divided by the slope.* | |
| Dividing the y-axis intersecion by the slope including minus sign (if minus sign is forgotten, i.e. the value is negative, -0.5 pt). | 1 |
| Value between 0.95 and 1.15 (in principle the value should be 1, however due to imprecisions a value slightly higher than one is more likely obtained). The value obtained in the measurement above is 1.03. | 1 |
| Interpretation: each $C_i$ describes how the magnetic flux $\Phi_i$ is connected to the corresponding voltage $U_i$. This connection is only dependent on the coil and the frequency. Since the two coils are the same, we expect to obtain a value close to 1. | 2 |

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# Physics Olympiad

## Final Round

18 - 19 March 2023

| | | |
|---|---|---|
| **Part 4** | **:** | **6 short questions** |
| Duration | : | 60 minutes |
| Total | : | 24 points ($6 \times 4$) |
| Authorized material | : | Calculator without database |
| | | Writing and drawing material |

# Good luck!


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Swiss Physical Society
Università della Svizzera italiana
Universität Bern FB Physik/Astronomie
Universität Zürich FB Physik Mathematik

## Natural constants

| | | | | |
|---|---|---|---|---|
| Caesium hyperfine frequency | $\Delta\nu_{\mathrm{Cs}}$ | 9.192 631 770 | $\times 10^{9}$ | $\mathrm{s}^{-1}$ |
| Speed of light in vacuum | $c$ | 2.997 924 58 | $\times 10^{8}$ | $\mathrm{m \cdot s^{-1}}$ |
| Planck constant | $h$ | 6.626 070 15 | $\times 10^{-34}$ | $\mathrm{kg \cdot m^2 \cdot s^{-1}}$ |
| Elementary charge | $e$ | 1.602 176 634 | $\times 10^{-19}$ | $\mathrm{A \cdot s}$ |
| Boltzmann constant | $k_{\mathrm{B}}$ | 1.380 649 | $\times 10^{-23}$ | $\mathrm{K^{-1} \cdot kg \cdot m^2 \cdot s^{-2}}$ |
| Avogadro constant | $N_{\mathrm{A}}$ | 6.022 140 76 | $\times 10^{23}$ | $\mathrm{mol^{-1}}$ |
| Luminous efficacy of radiation | $K_{\mathrm{cd}}$ | 6.83 | $\times 10^{2}$ | $\mathrm{cd \cdot kg^{-1} \cdot m^{-2} \cdot s^3 \cdot sr}$ |
| Magnetic constant | $\mu_0$ | 1.256 637 062 12(19) | $\times 10^{-6}$ | $\mathrm{A^{-2} \cdot kg \cdot m \cdot s^{-2}}$ |
| Electric constant | $\varepsilon_0$ | 8.854 187 812 8(13) | $\times 10^{-12}$ | $\mathrm{A^2 \cdot kg^{-1} \cdot m^{-3} \cdot s^4}$ |
| Gas constant | $R$ | 8.314 462 618... | | $\mathrm{K^{-1} \cdot kg \cdot m^2 \cdot mol^{-1} \cdot s^{-2}}$ |
| Stefan-Boltzmann constant | $\sigma$ | 5.670 374 419... | $\times 10^{-8}$ | $\mathrm{K^{-4} \cdot kg \cdot s^{-3}}$ |
| Gravitational constant | $G$ | 6.674 30(15) | $\times 10^{-11}$ | $\mathrm{kg^{-1} \cdot m^3 \cdot s^{-2}}$ |
| Electron mass | $m_{\mathrm{e}}$ | 9.109 383 701 5(28) | $\times 10^{-31}$ | kg |
| Neutron mass | $m_{\mathrm{n}}$ | 1.674 927 498 04(95) | $\times 10^{-27}$ | kg |
| Proton mass | $m_{\mathrm{p}}$ | 1.672 621 923 69(51) | $\times 10^{-27}$ | kg |
| Standard acceleration of gravity | $g_{\mathrm{n}}$ | 9.806 65 | | $\mathrm{m \cdot s^{-2}}$ |

## Short questions

Duration: 60 minutes
Marks: 24 points ($6 \times 4$)

Start each problem on a new sheet in order to ease the correction.

### Short question 4.1: Hot wire *(4 points)*

**i. (4 pts)** A wire of length $l$ and constant radius $r \ll l$ is connected to a voltage source with voltage $V$. The resistivity of the wire $\rho$ is temperature-dependant: $\rho(T) = cT$ for some constant $c > 0$. The wire radiates like a black body. How much current flows through the wire at equilibrium?

### Short question 4.2: Maximal power *(4 points)*

**i. (4 pts)** We have a voltage source with an internal resistance of $R_{\mathrm{i}} = 50\,\Omega$. What resistance $R$ do we have to connect to the voltage source, such that we have maximal heat dissipation over $R$?

### Short question 4.3: Taking a picture of a picture *(4 points)*

When taking a photo of an object, the lens (or a lens system) projects an image of the object onto the camera sensor. Assuming we reduce the distance between the lens and the sensor, without adjusting the focal length of the lens, the sharp image will be behind the sensor and it will only capture a fuzzy image (we assume the resolution is sufficiently high and does not need to be considered), see Figure 1. Can we print this blurred image and photograph it again with a camera, so that if the sensor is placed appropriately (with respect to the lens), a sharp image is obtained?

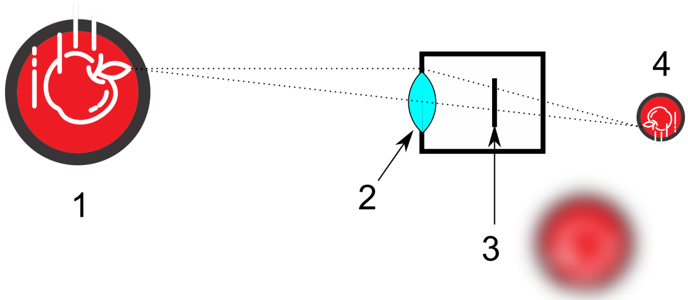



Figure 1: Sketch of the arrangement. 1: object to be imaged, 2: lens, 3: sensor with example of blurred image, 4. actual image of the object (behind the chip).

**i. (1 pt)** Is it possible, yes or no?
**ii. (3 pts)** Justify your answer.

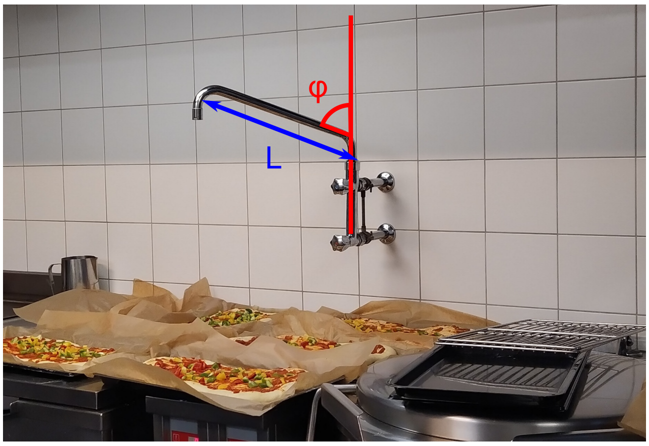
### Short question 4.4: Buoyancy *(4 points)*

A cylinder of height $L = 20\,\mathrm{cm}$ and radius $r = 50\,\mathrm{cm}$ has a homogeneous density $\rho_{\text{cylinder}}$.
Emmy puts the cylinder in water with one of the two circular faces pointing down. She marks the cylinder with a line where it touches the water surface. She then flips the cylinder upside down such that the other circular face is pointing down. She marks again the cylinder with a line where it touches the water surface. The two lines are located $d = 4\,\mathrm{cm}$ away from each other.
**i. (4 pts)** Compute the density of the cylinder. Assume that $\rho_{\text{air}} \ll \rho_{\text{water}} = 1000\,\mathrm{kg \cdot m^{-3}}$.

## Short question 4.5: Waving a faucet *(4 points)*

During the physics camp, the kitchen team noticed that a lot of water flowed out of the faucet when waving it, even though the faucet was closed. Why is that? We suppose the following parameters:

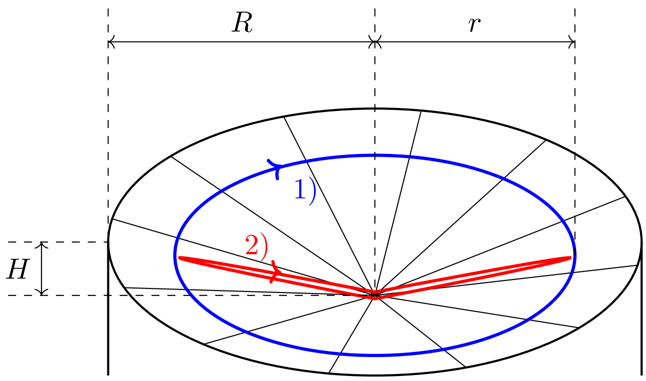
- The faucet is not horizontal, but slightly inclined with an angle of $\varphi = 70°$ with respect to the vertical rotational axis.
- The length of the inclined part of the faucet is $L = 50\,\text{cm}$.
- The faucet is closed, but the entire lenght $L$ is filled with water.
- The diameter of the pipe is $D = 1\,\text{cm}$.




**i. (4 pts)** Calculate the angular velocity $\omega$ around the rotational axis which is needed such that half of the water in the inclined part of the tube flows out.

## Short question 4.6: The Munot *(4 points)*

The Munot is a fortified tower in the city of Schaffhausen. It has a cylindrical shape with a radius $R = 20\,\text{m}$. Its top is arranged as a platform with a slight depression, with the shape of an inverted cone coaxial to the tower and a depth $H$ (in fact, a grid is located at the center to drain rainwater but we do not take it into account here). We also neglect friction forces.




We would like to measure $H$, but we only have a ball (with a negligible radius) and a stopwatch. We stand on the platform at a horizontal distance $r < R$ from the center, and proceed as following: [a]

- First, we roll the ball tangentially to the cone with an initial velocity such that it follows a circle at constant altitude until it comes back to us. We measure a duration $T_1$ for this trajectory.
- Then, we let the ball roll with zero initial velocity. Thus, it goes down to the center of the cone, goes up on the other side, stops, and comes back along the same path until it comes back to us. We measure a duration $T_2$ for this trajectory.

**i. (4 pts)** We find $\frac{T_1}{T_2} = 1.105$. Compute $H$.

[a] Note: nothing indicates that this measurement method is practically relevant...

# Short questions: solutions

## Short question 4.1: Hot wire — 4

**i. A wire of length $l$ and constant radius $r \ll l$ is connected to a voltage source with voltage $V$. The resistivity of the wire $\rho$ is temperature-dependant: $\rho(T) = cT$ for some constant $c > 0$. The wire radiates like a black body. How much current flows through the wire at equilibrium?** *4*

*The resistance is given by $R(T) = \frac{l}{r^2\pi}\rho(T) = \frac{l}{r^2\pi}cT$. The heating power is given by $P_{heat} = \frac{V^2}{R(T)}$. The black body radiation is given by the Stefan-Boltzmann law $P_{rad} = \sigma T^4 A$, where $\sigma$ is the Stefan-Boltzmann constant and $A = 2\pi rl$ is the surface of the wire. At equilibrium, $P_{heat} = P_{rad}$, leading to:*

$$\begin{aligned} \frac{V^2}{R(T_e)} &= \sigma T_e^4 A \\ \frac{V^2 r^2 \pi}{lcT} &= \sigma T_e^4 2\pi rl \\ \frac{V^2 r}{2\sigma l^2 c} &= T_e^5 \\ T_e &= \left(\frac{V^2 r}{2\sigma l^2 c}\right)^{\frac{1}{5}} \end{aligned}$$

*Using $I_e = \frac{V}{R(T_e)}$ we find:*

$$I_e = V^{\frac{3}{5}} \frac{(2\sigma)^{\frac{1}{5}}\, r^{\frac{9}{5}}\pi}{l^{\frac{3}{5}} c^{\frac{4}{5}}}$$

| | |
|---|---|
| Resistance $R(T) = \frac{l}{r^2\pi}\rho(T) = \frac{l}{r^2\pi}cT$. | 0.5 |
| Heating power $P_{\text{heat}} = \frac{V^2}{R(T)}$. | 0.5 |
| Stefan-Boltzmann law $P_{\text{rad}} = \sigma T^4 A$. | 0.5 |
| Area for the Stefan-Boltzmann law $A = 2\pi rl$ ($A = 2\pi rl + 2r^2\pi$ is also correct). | 0.5 |
| Equilibrium condition $P_{\text{heat}} = P_{\text{rad}}$. | 0.5 |
| Correct equilibrium temperature $T_{\text{e}} = \left(\frac{V^2 r}{2\sigma l^2 c}\right)^{\frac{1}{5}}$. | 0.5 |
| Correct equilibrium current. | 1 |
| *Points are also awarded if the equations were used implicitly.* | |

## Short question 4.2: Maximal power 4

**i. We have a voltage source with an internal resistance of $R_\mathrm{i} = 50\,\Omega$. What resistance $R$ do we have to connect to the voltage source, such that we have maximal heat dissipation over $R$?** 4

Let $U$ be the electromotive force at the voltage source then the current at resistance $R$ is

$$I = \frac{U}{R_\mathrm{i} + R},$$

1

which means the dissipated power over $R$ is

$$P = RI^2 = \frac{U^2}{(R_\mathrm{i} + R)^2} R.$$

1

Maximal power means that

$$0 = \frac{\partial P}{\partial R} = \frac{U^2 \left( (R_\mathrm{i} + R)^2 - 2R\,(R_\mathrm{i} + R) \right)}{(R_\mathrm{i} + R)^4},$$

1

which is fullfilled for $R = R_\mathrm{i} = 50\,\Omega$. 1

## Short question 4.3: Taking a picture of a picture *4*

**When taking a photo of an object, the lens (or a lens system) projects an image of the object onto the camera sensor. Assuming we reduce the distance between the lens and the sensor, without adjusting the focal length of the lens, the sharp image will be behind the sensor and it will only capture a fuzzy image (we assume the resolution is sufficiently high and does not need to be considered), see Figure 1. Can we print this blurred image and photograph it again with a camera, so that if the sensor is placed appropriately (with respect to the lens), a sharp image is obtained?**

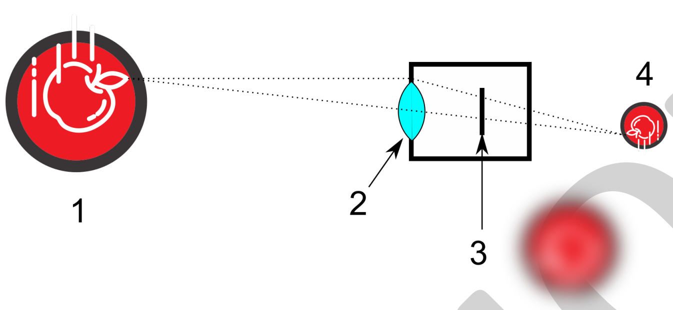



Figure 1: Sketch of the arrangement. 1: object to be imaged, 2: lens, 3: sensor with example of blurred image, 4. actual image of the object (behind the chip).



| | |
|---|---|
| **i. Is it possible, yes or no?** | *1* |
| *A "no" without justification is fine here.* | |
| No. | 1 |
| **ii. Justify your answer.** | *3* |
| *The reason why this does not work is the following: the camera records the intensity distribution. However, a sharp image is a coherent superposition of the field (electric and magnetic field) that is imaged by the lens. In more detail: the field distribution at the image is the same as at the object. Therefore the phase of the field is very important but not recorded. Such a crucial "information" contained in the field gets lost when only the intensity is recorded.* *The students do not need to answer in such detail, but some milestones should be mentioned (be generous in distributing the points, in particular if the description is a bit fuzzy).* | |
| The camera records an intensity. | 1 |
| Sharp image is superposition of waves from the object (main focus on superposition of field). | 1 |
| Phase information of superposition is lost. | 1 |

## Short question 4.4: Buoyancy *4*

**A cylinder of height $L = 20\,\text{cm}$ and radius $r = 50\,\text{cm}$ has a homogeneous density $\rho_{\text{cylinder}}$.**

**Emmy puts the cylinder in water with one of the two circular faces pointing down. She marks the cylinder with a line where it touches the water surface. She then flips the cylinder upside down such that the other circular face is pointing down. She marks again the cylinder with a line where it touches the water surface. The two lines are located $d = 4\,\text{cm}$ away from each other.**

**i. Compute the density of the cylinder. Assume that $\rho_{\text{air}} \ll \rho_{\text{water}} = 1000\,\text{kg}\cdot\text{m}^{-3}$.** *4*

There are two possibilities for the density of the cylinder, as the cylinder can sink by $\frac{L+d}{2}$ or $\frac{L-d}{2}$. (0.25 point for each correct expression of the sinking length and 0.75 point for recognizing the existence of two solutions. No point reduction in later parts for giving only one solution.) 1.25

The weight of the displaced water is

$$\frac{L \pm d}{2}\pi r^2 \rho_{\text{water}} g.$$

0.25

The weight of the cylinder is

$$L\pi r^2 \rho_{\text{cylinder}} g.$$

0.25

Using Archimedes' principle, we can equate both expressions. 0.5

This gives

$$\frac{L \pm d}{2}\rho_{\text{water}} g = L\rho_{\text{cylinder; 1, 2}} g.$$

0.5

Solving for $\rho_{\text{cylinder}}$:

$$\rho_{\text{cylinder; 1, 2}} = \frac{L \pm d}{2L}\rho_{\text{water}}.$$

(-0.5 point for small errors) 0.75

Numerically:

$$\rho_{\text{cylinder; 1}} = 600\,\text{kg}\cdot\text{m}^{-3},$$
$$\rho_{\text{cylinder; 2}} = 400\,\text{kg}\cdot\text{m}^{-3}.$$

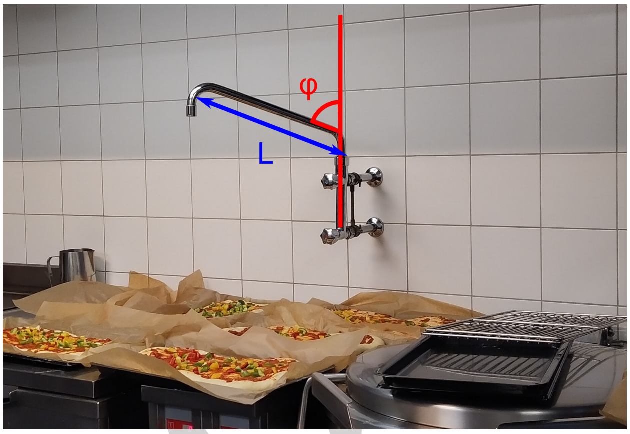
0.5

*One could also take the mass of displaced air into account at first. Setting $\rho_{air}$ small then leads to the wanted formula.*

## Short question 4.5: Waving a faucet *4*

**During the physics camp, the kitchen team noticed that a lot of water flowed out of the faucet when waving it, even though the faucet was closed. Why is that? We suppose the following parameters:**

- **The faucet is not horizontal, but slightly inclined with an angle of $\varphi = 70°$ with respect to the vertical rotational axis.**
- **The length of the inclined part of the faucet is $L = 50\,\mathrm{cm}$.**
- **The faucet is closed, but the entire lenght $L$ is filled with water.**
- **The diameter of the pipe is $D = 1\,\mathrm{cm}$.**




**i. Calculate the angular velocity $\omega$ around the rotational axis which is needed such that half of the water in the inclined part of the tube flows out.** *4*

*The water faucet is rotated with an angular velocity $\omega$. Depending on the distance $r$ from the rotation axis, gravity might or might not be sufficient to compensate for the centripetal force due to the rotation.*

For a given distance $r$ of the water faucet with respect to the rotation axis, the needed centripetal acceleration to keep the water in place is given by

$$a_Z(r) = \omega^2 r.$$

0.5

Due to the slope, the projection of this centripetal force along the faucet, is given by $a_{Z,p}(r) = \cos\left(\frac{\pi}{2} - \varphi\right) a_Z(r) = \sin(\varphi)\, a_Z(r)$. 0.5

The gravitational acceleration pushing the water down along the faucet is given by $a_{G,p} = g\cos(\varphi)$. 1

We want the two accelerations to be the same at $x = L/2$. 0.5

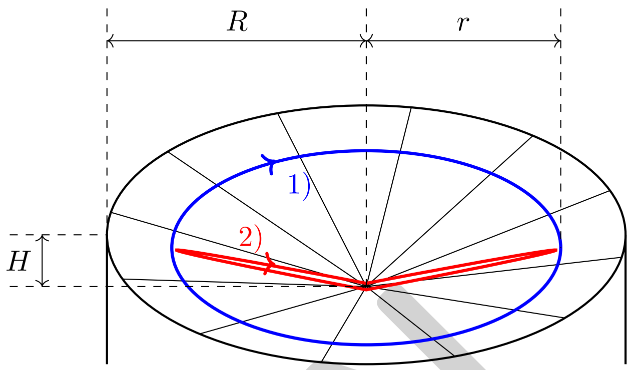
The distance along the faucet and the distance to the rotation axis are connected as $r = x\sin(\varphi)$. 0.5

Solving for $\omega = \sqrt{\frac{2g}{L\sin(\varphi)\tan(\varphi)}}$. 0.5

Numeric value of $3.90\,\mathrm{rad}\cdot\mathrm{s}^{-1}$. 0.5

## Short question 4.6: The Munot 4

**The Munot is a fortified tower in the city of Schaffhausen. It has a cylindrical shape with a radius $R = 20\,\mathrm{m}$. Its top is arranged as a platform with a slight depression, with the shape of an inverted cone coaxial to the tower and a depth $H$ (in fact, a grid is located at the center to drain rainwater but we do not take it into account here). We also neglect friction forces.**




**We would like to measure $H$, but we only have a ball (with a negligible radius) and a stopwatch. We stand on the platform at a horizontal distance $r < R$ from the center, and proceed as following:** [1]

- **First, we roll the ball tangentially to the cone with an initial velocity such that it follows a circle at constant altitude until it comes back to us. We measure a duration $T_1$ for this trajectory.**
- **Then, we let the ball roll with zero initial velocity. Thus, it goes down to the center of the cone, goes up on the other side, stops, and comes back along the same path until it comes back to us. We measure a duration $T_2$ for this trajectory.**

**i. We find $\frac{T_1}{T_2} = 1.105$. Compute $H$.** 4

The depth of the surface from our position is $h$, with

$$\frac{r}{h} = \frac{R}{H}.$$

0.25

In the first case, we have a uniform circular motion obeying the relation

$$a_1 = \frac{v^2}{r}.$$

0.25

The acceleration can be found by projecting the forces (weight and normal force), knowing that it is radial and horizontal. We find

$$\frac{a_1}{g} = \frac{h}{r}.$$

[1]Note: nothing indicates that this measurement method is practically relevant...

0.5

The velocity is related to $T_1$ by

$$v = \frac{2\pi r}{T_1}.$$

0.25

*Putting it together we find*

$$T_1 = 2\pi \frac{r}{\sqrt{gh}}.$$

For the second case, we have a piecewise uniformly accelerated motion. Due to spatial symmetry and temporal symmetry (energy conservation), it is sufficient to find the rolling time from the initial position to the center, then multiply by four. 0.25

As in the first case, the acceleration points towards the axis, but this time tangentially to the slope. By projecting along the slope:

$$a_2 = g\frac{h}{\sqrt{r^2 + h^2}}.$$

0.5

The time obeys

$$\sqrt{r^2 + h^2} = \frac{1}{2}a_2 \left(\frac{T_2}{4}\right)^2.$$

0.5

*We therefore find*

$$T_2 = 4\sqrt{2}\frac{\sqrt{r^2 + h^2}}{\sqrt{gh}}.$$

The time fraction is thus

$$\begin{aligned}\frac{T_1}{T_2} &= \frac{\pi\sqrt{2}}{4}\frac{r}{\sqrt{r^2 + h^2}}\\ &= \frac{\pi\sqrt{2}}{4}\frac{1}{\sqrt{1 + \left(\frac{h}{r}\right)^2}}\\ &= \frac{\pi\sqrt{2}}{4}\frac{1}{\sqrt{1 + \left(\frac{H}{R}\right)^2}}.\end{aligned}$$

As we see, only the slope (the fraction $\frac{h}{r}$) plays a role, not the actual values of $h$ and $r$, we can therefore replace them by $H$ and $R$.
Inverting for $H$ we find

$$H = R\sqrt{\frac{\pi^2}{8\left(\frac{T_1}{T_2}\right)^2} - 1}.$$

1

Numerically,

$$H \approx 2.038\,\mathrm{m}.$$

0.5

***Alternative solution:*** *(4)*

*We can use the angle* $\alpha = \arctan(H/R)$*:*

*For the circular motion, we get*

$$a_1 = \omega^2 r,$$

*where*

$$\omega = \frac{2\pi}{T_1}$$

(0.5)

*and*

$$a_1 = g\tan(\alpha)\,,$$

(0.25)

*which gives*

$$g\tan(\alpha) = r\frac{4\pi^2}{T_1^2}$$

*and solving for* $r$*:*

$$r = g\tan(\alpha)\,\frac{T_1^2}{4\pi^2}.$$

(0.5)

*For the linear motion, we again (as in the other solution) use only the first quarter. Since the acceleration is constant, we can use*

$$x = \frac{1}{2}a_2 t^2,$$

(0.25)

*where*

$$x = \frac{r}{\cos(\alpha)},$$

$$a_2 = g\sin(\alpha)\,,$$

$$t = \frac{T_2}{4}.$$

(0.5)

*This gives*

$$\frac{r}{\cos(\alpha)} = g\sin(\alpha)\,\frac{T_2^2}{32}$$

*and solving again for* $r$*:*

$$r = \cos(\alpha)\,g\sin(\alpha)\,\frac{T_2^2}{32}.$$

(0.5)

*Putting the two equations for* $r$ *together, we get:*

$$g\tan(\alpha)\,\frac{T_1^2}{4\pi^2} = \cos(\alpha)\,g\sin(\alpha)\,\frac{T_2^2}{32}.$$

*Solving for* $\cos(\alpha)$*:*

$$\cos(\alpha) = \sqrt{\frac{8}{\pi^2}\left(\frac{T_1}{T_2}\right)^2} = \frac{2\sqrt{2}}{\pi}\frac{T_1}{T_2}.$$

*Finally, we use* $H = R\tan(\alpha)$ *to obtain*

$$H = R\tan\left(\arccos\left(\frac{2\sqrt{2}}{\pi}\frac{T_1}{T_2}\right)\right).$$

(1)

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*Numerically,*

$$H \approx 2.038\,\mathrm{m}.$$

(0.5)

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