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Swiss Physics Olympiad 2022

Aarau, March 19th 2022

Experiment 2
Helmholtz Resonator

Duration: 90 minutes
Total: 24 points

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Allowed aids Calculator without formulae memory
writing and drawing material

Remark2. This experiment was prepared already by mid of March 2020 an replaced by an online experiment due to corona.

Content

PHYSICS OLYMPIAD is supported by

Staatssekretariat für Bildung, Forschung und Innovation
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Deutschschweizerische Physikkommission VSMP / DPK
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Ecole Polytechnique Fédérale de Lausanne
ETH Zurich Department of Physics
Fondation Claude & Giuliana
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Swiss Academy of Engineering Sciences SATW
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Università della Svizzera italiana
Universität Bern FB Physik/Astronomie
Universität Zürich FB Physik Mathematik

1. Introduction

Surely you know the following phenomenon: by blowing over the opening of an empty bottle a sound can be produced. One might think that this way standing waves in the bottle are generated, similar as in an organ pipe. However, if one measures the frequency and compares it with that of standing waves in the bottle, one finds that this explanation cannot be true. Hermann von Helmholtz gave the right explanation for these vibrations just over 160 years ago.

Helmholtz resonators

Helmholtz explained the vibrations in this way (see Fig. 1): The air column in the bottleneck H acts as an (inertial) mass in the presence of vibrations: the cylindrical air mass moves back and forth as a whole. The elasticity of the air volume V_L acts on the other hand like a spring, constituting a mass-spring system. This division in mass and 'spring' is justified as follows: During the vibrations, the air in the bottleneck moves significantly faster than the air mass in the bottle volume due to the continuity equation. Correspondingly, the accelerations are larger, and thus the inertial mass of the air in the bottleneck starts to outweigh. Conversely, the air in the bottle volume moves more slowly than in the neck and acts as an 'air cushion' and thus as a spring. Such resonators are called Helmholtz resonators. Note: In a Helmholtz resonator, the resonance frequency is not given by that of a standing wave like in an organ pipe!

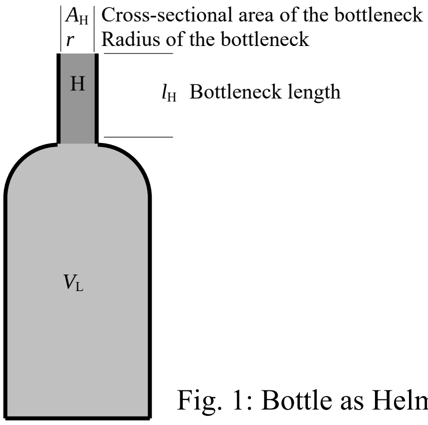


Fig. 1: Bottle as Helmholtz resonator

An analysis of the problem shows that the resonance frequency depends on the dimensions as follows:

$$f = \frac{b}{2\pi}(V_L)^{\alpha}\,(l'_H)^{\beta}\,(A_H)^{\gamma} \tag{1}$$

The length l'_H is the so-called effective bottleneck length

$$l'_H = l_H + 1.6 \cdot r. \tag{2}$$

The physical reason for this is that the movement of the vibrating air column in the tubular neck does not end abruptly at the ends, but has a transition zone in open space. This must be added to the inertial mass.

2. Material

1 bottle (PET beverage bottle)
3 thick overhead projector transparencies
scissors
tape
ruler
caliper
sufficient water supply in PET bottles
funnel, household paper
scale
tripod foot
evaluation paper sheets
frequency generator, BNC cable
oscilloscope (digital)
microphone (powered via USB)

3. Hints (read thoroughly before conducting the experiment)

The solution must contain:

- A brief and clear description of how the measurement was done and what was taken into account
- Clear presentation of the measurement results
- Reasoning why the specific method used for evaluating the measurement results was chosen
- Documented evaluation

The resonance frequencies are measured by excitation with a sine signal produced by a loudspeaker near the bottle opening. The speaker is driven by a sound generator. The sound amplitude is recorded with a small microphone in the bottle volume V_L and is displayed with an oscilloscope (in arbitrary units). For resonance the frequency is set in such a way that a maximum amplitude is reached. Make sure the microphone never is dipped into the water. Details and important hints on the sound generator can be found in Appendix A, for the oscilloscope in Appendix B, and for the microphone in Appendix C.

Physical constants

Speed of sound c = 340 m/s
Density of water ρ = 1000 kg/m^3

4. Tasks

(a) Measurements on the empty bottle.

(a1) Try to produce a sound by blowing onto the empty bottle and measure its frequency f_1. The microphone must be outside of the bottle for the measurement of the frequency.

(a2) Calculate the frequency f_2 under the assumption that a standing wave in the bottle is forming.

(b) Determination of the exponent α.

Study the resonance frequency f for different volumes V_L (typically 5 to 7 different measurements) at constant bottle neck length l_H. Do not use volumes smaller than 0.20 liters.

Determine the exponent α through a suitable data analysis method.

(c) Determination of the exponent β.

Study the resonance frequency f for various bottle length necks l. You will have to typically conduct 4 to 7 measurements. The measurements for different bottleneck lengths should be conducted with a bottle volume of approx. 0.7 liters.
By an appropriate evaluation method, determine the exponent β from the measurement data.

Note:
The neck length can be extended with the enclosed transparencies as follows: A suitable piece of transparency is rolled and pushed into the bottleneck. You can cut the transparency with the scissors if necessary and fix it with tape. The shape of the pipe produced in this way should be cylindrical, not conical. Always use the same number of windings of the transparency (reason: this way the bottleneck cross section is always the same). That is why it is also important that the transparency is fully inserted all the way into the bottleneck. But do not push the transparency in too far, since you have to be able to pull it out again!

(d) Determination of constants *b*

The third exponent γ in formula (1) cannot be measured by simple means, which is why it is given here:

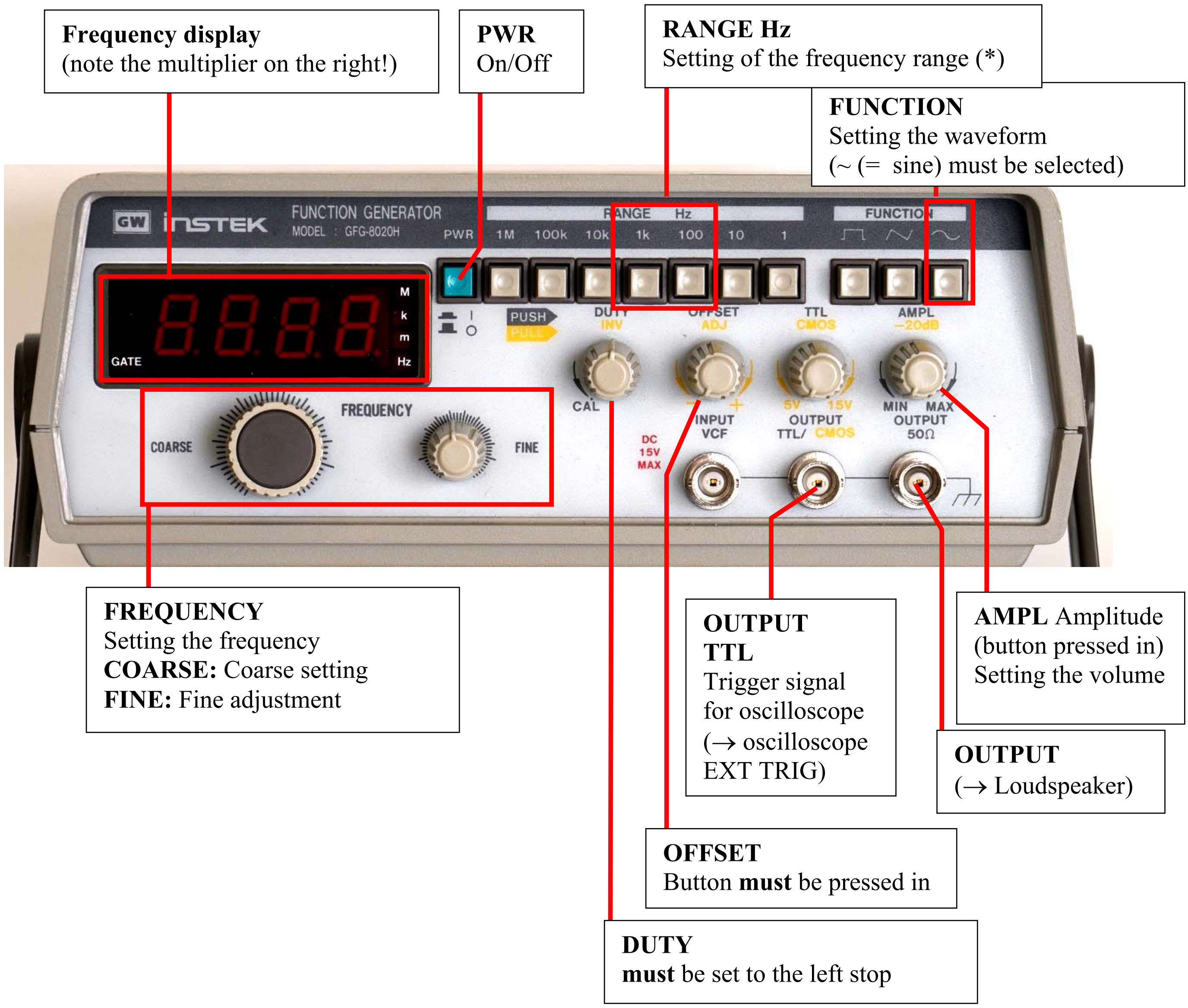
$$f \text{ is proportional to } A_H^{1/2} \quad (\gamma \text{ is } = ½). \tag{3}$$

Determine the constant b of the expression (1) using the results obtained in (b) and (c). For the calculation using formula (1) use the maximum air volume and the frequency f_1 from task (a1), and equation (3) taking into account the size of the bottleneck.

Function generator

Use:
1. Generating a sine signal with an adjustable frequency and amplitude
2. Supplying this signal to the loudspeaker

Brief description of the most important buttons:



(*) Setting range for the frequency: (button FINE in middle position)

Position "100":	approx. from 16 Hz to 230 Hz
Position "1k"	approx. from 160 Hz to 2.3 kHz

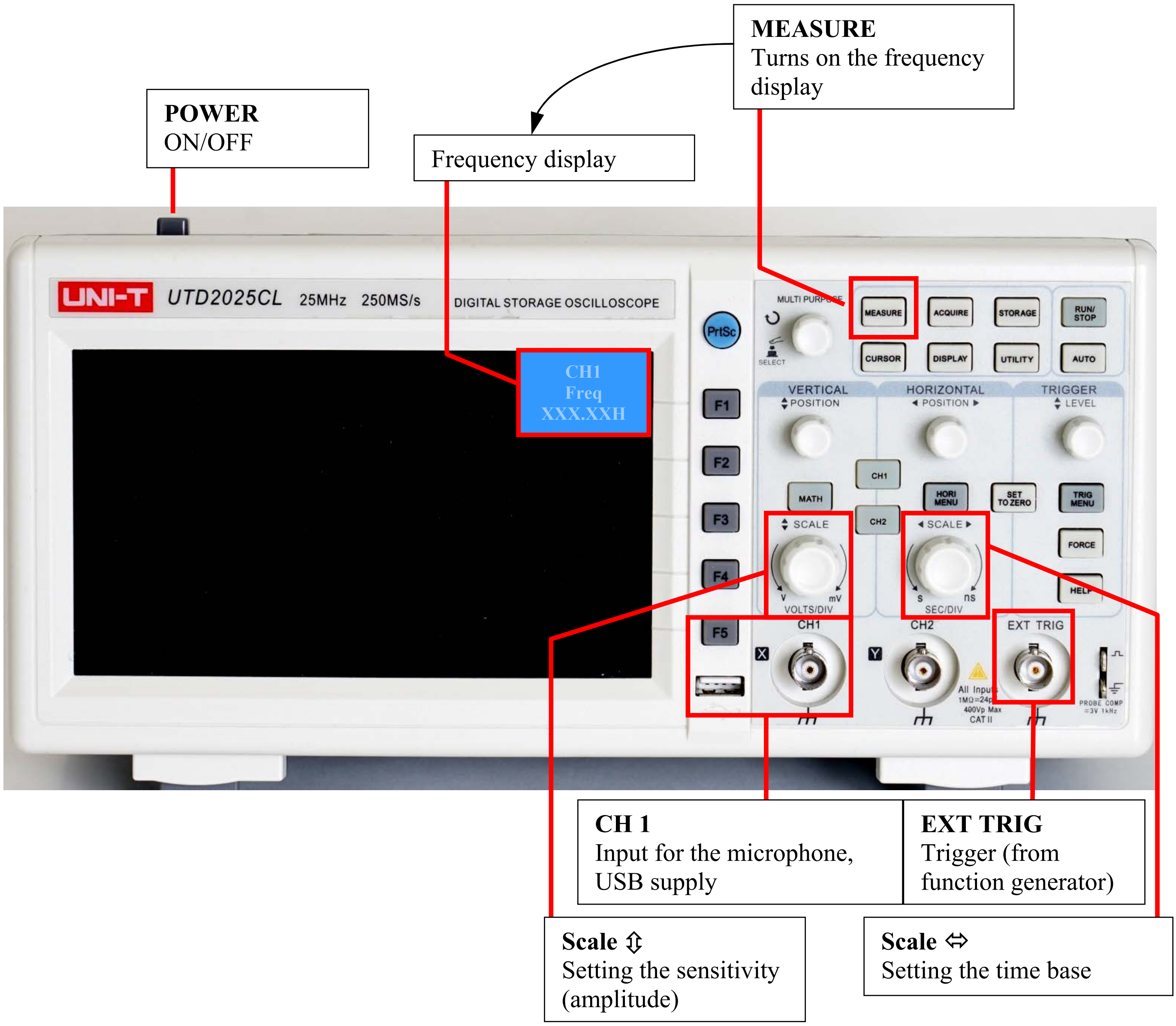
The frequency display typically fluctuates in the rearmost digit. The reading can be based on typically 3 significant digits.

Always use the lowest possible frequency range RANGE (reason: achieving the optimal resolution when setting the frequency).

Digital oscilloscope

Use:

1. Displaying the microphone signal (determination of the maximum amplitude at resonance)
2. Measurement of the frequency of the microphone signal (task (a1))

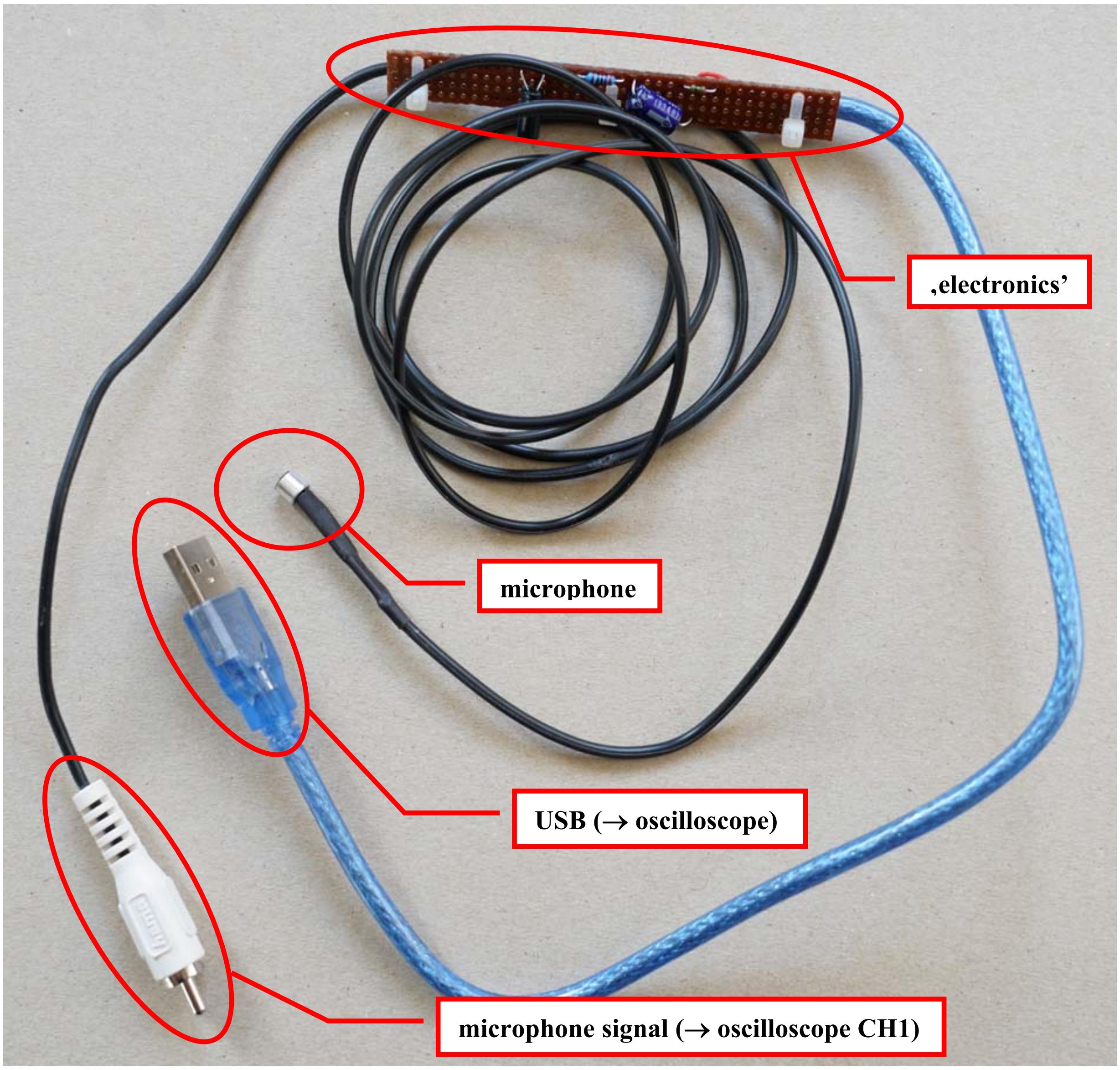


Comments

- The cable connections to the microphone and function generator have already been made.
- On the microphone signal disturbances may be visible, especially with weak signals. However, these do not interfere with the determination of the resonance frequency.
- For the frequency measurement (task (a1)) it is necessary that the representation of the signals is not too small and not too large (overload). This is achieved by setting **SCALE** ⇳ appropriately. A few oscillations should be visible on the screen, adjusted by **SCALE** ⬄ **.**
- The MEASURE button turns on the frequency measurement on the screen. This reading may differ slightly from the frequency of the function generator (relative error < 10^{-3} or 10^{-4})
- The oscilloscope is triggered externally to ensure that the signal on the screen is stable. The trigger signal comes from the function generator (OUTPUT TTL).
- Using the external trigger feature of the oscilloscope, you will be able to see a clear phase shift of the microphone signal close to the resonance frequency.

Appendix C Microphone with USB connection

The image shows the microphone with the connection cables. The microphone will already be properly connected at your workplace.



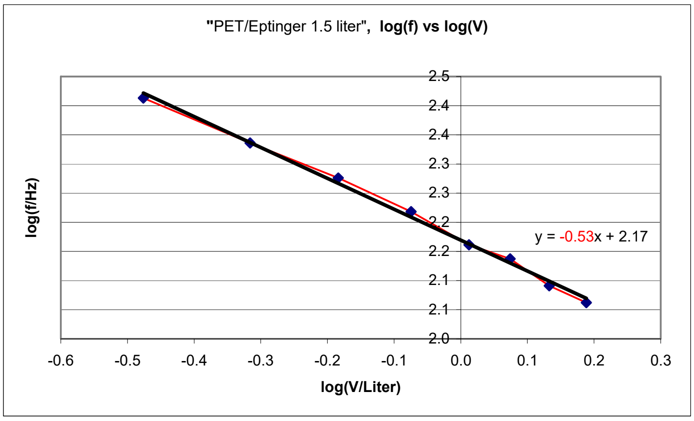
EXPERIMENT 2 Solution/Marking Scheme

task	subtasks	POINTS
(a)	**Measurements of the empty bottle (note: in the sample solution a 1.5 litre PET bottle was used)**	**(3)**
	Measurement of the frequency of the empty bottle with the blowing method • Microphone outside the bottle (given in the task) • Frequency f_1 = 117 Hz (within range (117 -5) Hz to (117 + 5) Hz) (value for a 1.5 litre PET bottle) (frequency depends on the avtual bottle used) Value should be given as **XXX Hz** (and not as XXX.X Hz oder XXX.XX Hz): reasonable accuracy **[-1/2 if not]**	1
(a2)	**Calculation of the standing frequency f_2 in the bottle** • Standing wave in the bottle, one end closed, one end open: $\lambda/4 = h$ (height of the bottle), $\rightarrow \boldsymbol{\lambda = 4 \cdot h}$ • $f_2 = c/\lambda = \mathbf{c/(4 \cdot h)}$ **[0 pts if h is not equal to $\lambda/4$]**	1
	• numerical values **f_2 from 340/(4·0.34) = 250 Hz** to **f_2 = 340/(4·0.31) = 274 Hz** acceptable values of height h: without bottleneck: 310 mm, with bottleneck 340 mm **[–1 pts if numerical value wrong, –1/2 if units are wrong]**	1
(b)	**Determination of exponent α (for sample data and graph see next page (b'))**	**(8)**
	Measurements • **determination of bottle air volume (guessed values like 1.5 l are not accepted! The bottle has no volume marking)** most precise method: measure the weiht of the bottle, fill the bottle up to the neck with water, weight the water (or tara with bottle) **[method measured *height × cross section* is not accepted: 0 pts]** measured values: empty bottle: 32 - 33 g, with water: 1590 g ± 5 g, water only: **1557 g ± 7 g, Volume 1.557 l** (the students are free in the choice of the units, however, a clear statement of the units is mandatory)	1
	• **description of experimental set up/method** determination of air volume by weighting method (correct calculations, reproducible) loudspeaker approx. 1 cm to some cm above the opening of the bottle, attached with adhesive tape to the stand (sketch) microphone inserted in the bottle, in the air volume	1
	• **data** (for sample values see reverse side) 5+ measured values, clearly arranged in a table **[if less than 5 measured points: 1 pt., less than 3: 0 pts.]** points approx. equally distributed over volume range ~1.557 l to 0.15 l frequency should increase for decreasing volume **[plausibility check for measured frequencies: see next page]**	2
	Data evaluation (for sample graph see reverse side) • The only possible method for the determination of α is a log-log plot (α = slope) • Plot of the data: abscissa: log(V), ordinate: log(f) if student uses reversed axes: check for correct sign of slope plot either by using the log/log paper or by calculation of log(V) and log(f) and a lin/lin plot (aequivalent) **[method]** The used units are irrelevant (e.g. m^3, cm^3, mm^3, l) correct plot required with labelled axes, best straight line determination of the slope (used triangle should be indicated) **[axes not labelled: –1/2 pt]** • Correct result (**slope approx –0.5**, – sign important) **[wrong sign: –1/2 pt]**	1 2 1
(c)	**Determination of exponent β (for sample data and graph see next page (c'))**	**(9)**
	Experimental method (description) • roll the given foil to a tube, insert the tube correctly (inner end of the tube at the inner end of the bottle throat) use adhesive tape to stabilise the arrangement: sketch with setup and measured length of throat • for resonance frequency determination use the analogue method as in (a) (speaker/microphone arrangement) • bottle should be filled with approx 0.8 l of water (for an air volume of ~0.7 l), exact value is not important (the water serves to shift the resonance frequencies to higher values: better efficiency of loudspeaker an microphone)	1
	measurements (for sample values see reverse side, due to inaccurate water volume and throat diameter comparison is difficult) • use as much of the full possible range 30 cm **[0 pts if length smaller than 20 cm]** • ~equally spaced length like 5cm / 10 cm / 15 cm / 20 cm / 25 cm / 30 cm resulting in 6 measured values (not less than 5) **[1/2 pt if 5 lengths, 0 pts if less than 4 lengths]** • frequency should decrease for increasing throat length (check for grade) • measured values clearly arranged in a table (with units) **[–1/2 pts for missing units]**	1 1 1
	• **Data evaluation** (for sample graph see reverse side) • The only possible method for the determination of β is a log-log plot (β = slope) • End correction (extension of the table): the length should be corrected $l' = l + 1.6 \cdot r$ (*) throat radius r = **1.0.4 cm** (for 3 turns of the foil) **[1/2 pt if end correction wrong]** • Plot of the data: abscissa: log(***effective*** *throat length*), ordinate: log(f) if student uses reversed axes: check for correct sign of slope plot either by using the log/log paper or by calculation of log(*effective throat length*) and log(f) and a lin/lin plot **[method]** the used units are irrelevant (m, cm, mm) correct plot with labelled axes, best straight line determination of the slope (used triangle should be indicated) **[axes not labelled: –1/2 pt]** • Correct result (slope approx –0.5, – sign important) **[wrong sign: –1/2 pt]**	1 1 2 1
(d)	**Determination of constant b (for sample data see next page (d'))**	**(4)**
	Solve $f = \frac{b}{2\pi}(V_L)^{\alpha}(l'_H)^{\beta}(A_H)^{\gamma}$ for b: $b = \frac{2\pi f}{(V_L)^{\alpha}(l'_H)^{\beta}(A_H)^{\gamma}}$	1
	• use the values V_L full air volume, **from (b),** corresponds to measurement of f_1 in (a) α **from (b)** l' throat length of bare bottle + end correction (**1.6**·r)!, **data from (b)** β **from (c)** $A_H = \pi r^2$ throat cross section (bare bottle without cable, corresponds to (a)), $\mathbf{r}$ **= 10.8 mm** γ given in the task (= ½) f_1 **from (a),** for full air volume, no cable, **[1 pt if one input data is wrong, 0 pts if two or more input data wrong]**	2
	• correct numerical evaluation of the given data (from theory b is the speed of sound!)	1

(b) Determination of exponent α: sample data and graphs

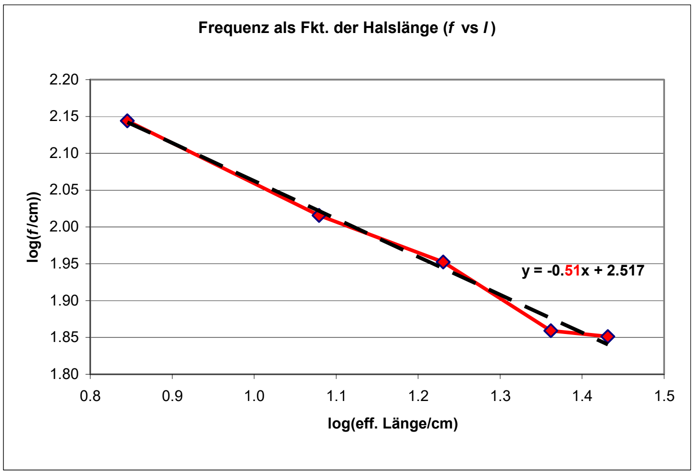
bottle volume 1.543 Liter

V_incr	Füllemenge	Vol	f/Hz	log(V(Liter)	log(f/Hz)	f_ratio
	0	**1.543**	**115.4**	0.188	2.062	1
0.185	0.185	**1.358**	**123.3**	0.133	2.091	1.07
0.172	0.357	**1.186**	**137.2**	0.074	2.137	1.19
0.157	0.514	**1.029**	**144.9**	0.012	2.161	1.26
0.187	0.701	**0.842**	**165.3**	-0.075	2.218	1.43
0.187	0.888	**0.655**	**188.8**	-0.184	2.276	1.64
0.172	1.06	**0.483**	**216.7**	-0.316	2.336	1.88
0.149	1.209	**0.334**	**259**	-0.476	2.413	2.24



(c) Sample values and data evaluation

Korrekturfaktor		2		
Radius Hals		1	cm	
Halslänge	**Halsl eff**	**f**	**log(l')**	**log(f)**
(cm)	(cm)	(Hz)	-	-
5	**7.0**	**139.4**	0.85	2.14
10	12.0	103.7	1.08	2.02
15	17.0	89.6	1.23	1.95
21	23.0	72.3	1.36	1.86
25	**27.0**	**71**	1.43	1.85



(d) Evaluation of constant *b*

Use the following data

Radius of throat $r = 1.08$ cm $= 0.0108$ m

Cross section of bottle throat $A_H = \pi r^2 = \pi\,(0.010)^2 = 0.000314\ \mathrm{m}^2$

Air volume (empty bottle) $V_H = 0.00150\ \mathrm{m}^3$

Frequency $f = 117$ Hz

Effective throat length $l'_H = l_H + 1.6\cdot r = 0.020\ \mathrm{m} + 1.6\cdot 0.010\ \mathrm{m} = 0.036\ \mathrm{m}$

Exponent $\alpha = -0.53$ from (b)

$\beta = -0.51$ from (c)

$\gamma = 0.50$ (given)

$$b = \frac{2\pi f}{(V_L)^{\alpha}\,(l'_H)^{\beta}\,(A_H)^{\gamma}} = \frac{2\pi f}{(V_L)^{-0.53}\,(l'_H)^{-0.51}\,(A_H)^{1/2}} = 250\ \mathrm{m/s}$$

According to the theory *b* is the speed of sound.

Many different measurements have shown, that the accuracy of *b* is not very good.

For the marking it is only important that the candidate solves the expression $f = \frac{b}{2\pi}(V_L)^{\alpha}\,(l'_H)^{\beta}\,(A_H)^{\gamma}$ for *b* and applies the correct values and units!

Remark: 1.(*) For the end correction ($l' = l + 1.6\cdot r$) different values can be found in the literature for the factor (1.6 is used)

2. This experiment was prepared already by mid of March 2020 replaced by another online experiment due to corona).