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Physics Olympiad

Second round

Bern, 15 January 2020

First part (60')	**:**	**MC – 22 questions**
Second part (120')	**:**	**Problems – 3 questions**
Authorized material	:	Calculator without database Writing and drawing material

Good luck!

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Natural constants

Speed of light in vacuum	c	$=$	$299\,792\,458\,\mathrm{m}\cdot\mathrm{s}^{-1}$
Vacuum permeability	μ_0	$=$	$4\pi\times10^{-7}\,\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{A}^{-2}\cdot\mathrm{s}^{-2}$
Vacuum permittivity	ε_0	$=$	$8.854\,187\,817\times10^{-12}\,\mathrm{A}^2\cdot\mathrm{s}^4\cdot\mathrm{kg}^{-1}\cdot\mathrm{m}^{-3}$
Planck constant	h	$=$	$6.626\,069\,57\times10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}$
Elementary charge	e	$=$	$1.602\,176\,565(35)\times10^{-19}\,\mathrm{A}\cdot\mathrm{s}$
Gravitational constant	G	$=$	$6.673\,84(80)\times10^{-11}\,\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$
Gravitational acceleration on Earth	g	$=$	$9.81\,\mathrm{m}\cdot\mathrm{s}^{-2}$
Avogadro constant	N_A	$=$	$6.022\,141\,29(27)\times10^{23}\,\mathrm{mol}^{-1}$
Universal gas constant	R	$=$	$8.314\,459\,8(48)\,\mathrm{J}\cdot\mathrm{mol}^{-1}\cdot\mathrm{K}^{-1}$
Boltzmann constant	k_B	$=$	$1.380\,648\,8(13)\times10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}$
Stefan-Boltzmann constant te	σ	$=$	$5.670\,373(21)\times10^{-8}\,\mathrm{W}\cdot\mathrm{m}^{-2}\cdot\mathrm{K}^{-4}$
Electron mass	m_e	$=$	$9.109\,382\,6(16)\times10^{-31}\,\mathrm{kg}$
Neutron mass	m_n	$=$	$1.674\,927\,28(29)\times10^{-27}\,\mathrm{kg}$
Proton mass	m_p	$=$	$1.672\,621\,71(29)\times10^{-27}\,\mathrm{kg}$

Multiple choice: answer sheet

Duration: 60 minutes
Marks: 22 points (1 point for each correct answer)
Indicate your answers in the corresponding boxes on this page.
For each question, there is only **one** correct answer.

Name :	
First name :	
Total :	

	a)	b)	c)	d)	e)	f)
Question 1	☐	☐	☐	☐		
Question 2	☐	☐	☐	☐	☐	☐
Question 3	☐	☐	☐	☐	☐	
Question 4	☐	☐	☐	☐	☐	
Question 5	☐	☐	☐	☐		
Question 6	☐	☐	☐	☐	☐	☐
Question 7	☐	☐	☐	☐		
Question 8	☐	☐	☐	☐	☐	☐
Question 9	☐	☐	☐	☐	☐	
Question 10	☐	☐	☐	☐	☐	
Question 11	☐	☐	☐	☐		
Question 12	☐	☐	☐	☐	☐	☐
Question 13	☐	☐	☐	☐	☐	☐
Question 14	☐	☐	☐	☐	☐	☐
Question 15	☐	☐	☐	☐	☐	☐
Question 16	☐	☐	☐	☐	☐	
Question 17	☐	☐	☐	☐	☐	☐
Question 18	☐	☐	☐	☐	☐	
Question 19	☐	☐	☐	☐	☐	
Question 20	☐	☐	☐	☐	☐	
Question 21	☐	☐	☐	☐	☐	
Question 22	☐	☐	☐	☐	☐	

Question 1

In this (hypothetical) scenario, our Sun has been replaced by a black hole with a mass equalling 1.5 times the mass of the Sun. Which statement about the Earth movement is correct?

a) The Earth trajectory does not change.

b) The Earth approaches the black hole in a spiral pathway and is then consumed.

c) The eccentricity increases.

d) Both semi-axes of the orbit grow significantly.

Question 2

A test mass is located a distance R away from the surface of a planet with radius R and mass M. The test mass has an inital velocity parallel to the planet's surface. What is the minimum required initial velocity to avoid the test mass crashing into the planet?

a) $\sqrt{\frac{GM}{3R}}$

b) $\sqrt{\frac{GM}{2R}}$

c) $\sqrt{\frac{2GM}{3R}}$

d) $\sqrt{\frac{3GM}{2R}}$

e) $\sqrt{\frac{GM}{R}}$

f) None of the other answers.

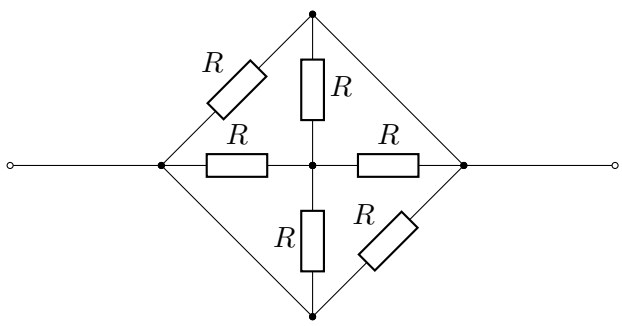
Question 3

Water flows vertically with an initial speed of $10\,\mathrm{cm}\cdot\mathrm{s}^{-1}$ from a tap with an opening of diameter 2 cm. What will be the diameter of the water jet 25 cm below the tap opening? The influence of pressure and surface tension can be neglected. *Remark: the exam originally contained an incorrect solution, which was corrected retrospectively.*

a) 0.4 cm b) 0.5 cm c) 0.8 cm d) 1.0 cm e) 2.0 cm

Question 4

The circuit shown below consists of six identical resistors R.



What is the total resistance of the system?

a) $\frac{R}{3}$ b) $\frac{R}{2}$ c) R d) $2R$ e) $3R$

Question 5

E stands for energy, T for temperature, v for velocity, m for mass, λ for distance, P for power and f for frequency. Which one of the following statements is false?

a) $\exp\left(\frac{E}{k_\mathrm{B}T}\right)$ b) $\log\left(\frac{v^3 m}{\lambda P}\right)$ c) $\sin(\lambda f)$ d) $\sqrt{1+\frac{c^2}{v^2}}$

Question 6

Given a distance d, a force F and a velocity v, what are the values of the constants α, β and γ which are needed in order for $d^\alpha F^\beta v^\gamma$ to have units of density?

a) $\alpha = -3$, $\beta = 1$, $\gamma = -2$

b) $\alpha = -3$, $\beta = -1$, $\gamma = 1$

c) $\alpha = -2$, $\beta = 1$, $\gamma = -2$

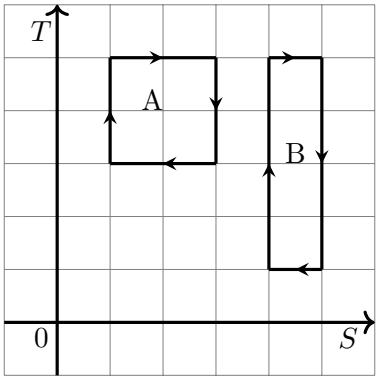
d) $\alpha = -3$, $\beta = 1$, $\gamma = -1$

e) $\alpha = -4$, $\beta = -1$, $\gamma = 2$

f) $\alpha = -2$, $\beta = -1$, $\gamma = -1$

Question 7

This diagram shows two cyclic processes of an ideal gas. Which statement is false?



a) The net work of the cycle A is the same as the one of cycle B.

b) The lowest temperature of cycle B is strictly smaller than the lowest temperature of cycle A.

c) The efficiency coefficient of cycle A is bigger than the one of cycle B.

d) The cycle A does not contain an isochoric process.

Question 8

Uranium-235, having a half-life of approximately 700 million years, decays to lead-207 (stable). The intermediary decay products have negligibly short half-life. In a sample of zircon (a crystal which doesn't contain lead when it forms), we measure the ratio of atoms to be $\frac{^{235}\mathrm{U}}{^{207}\mathrm{Pb}} = 0.1$. What is the lower limit on Earth's age that you can deduct from this measurement? ("a" ist the SI unit for a year)

a) 1.0 Ga

b) 1.5 Ga

c) 2.0 Ga

d) 2.4 Ga

e) 4.5 Ga

f) 5.1 Ga

Question 9

Let's consider a spring of negligible mass and a length at rest of 1 m. On Earth, we attach a mass of 1 kg and the spring's length increases by 1 m. If we let the same system oscillate vertically on the Moon, what would be the respective period of oscillation? The Moon's gravity is $1.6\,\mathrm{m}\cdot\mathrm{s}^{-2}$.

a) 0.3 s b) 2.0 s c) 5.0 s d) 6.1 s e) 12.3 s

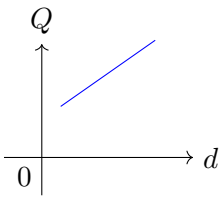
Question 10

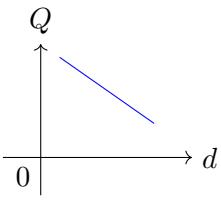
We consider a cylinder with radius r and height l as well as a cylindrical container with radius $2r$ and height $2l$. The latter is half-filled with a liquid of density ρ. We insert the cylinder vertically in the container. At equilibrium, the liquid's level moved up by $\frac{l}{6}$. What is the density of the cylinder?

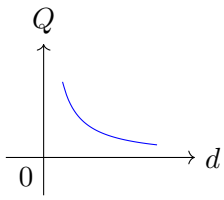
a) $\frac{2}{3}\rho$ b) $\frac{3}{4}\rho$ c) $\frac{4}{5}\rho$ d) $\frac{5}{6}\rho$ e) ρ

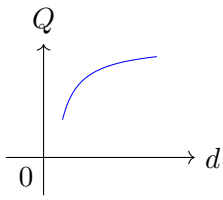
Question 11

The electrodes of a parallel plate capacitor are connected to a battery. When the distance d between the plates is varied, which graph best describes the charge Q stored in the positive capacitor plate?

a)


b)


c)


d)


Question 12

What is the best 3rd order polynomial approximation of $\sin(x)$ around the point $-\frac{\pi}{2}$?

a) $x - \frac{x^3}{6}$
b) $x + \frac{x^3}{6}$
c) $\sin(x)$
d) $\frac{\pi^2}{8} - 1 + \frac{\pi}{2}x + \frac{x^2}{2}$
e) $-1 + x^3$
f) -1

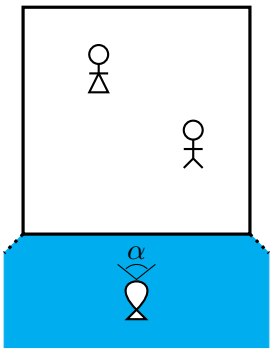
Question 13

One has a thin convex lens of focal length f. At which distance from the lens should you place an object to get a real upright image?

a) $\frac{f}{2}$
b) f
c) $\frac{3f}{2}$
d) $2f$
e) $\frac{5f}{2}$
f) None of those answers.

Question 14

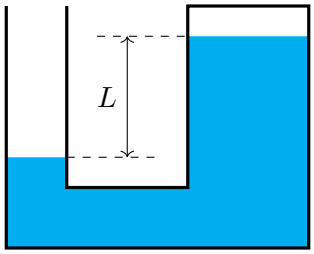
At the zoo, visitors are in a square room. One of the room's sides is the window of an aquarium filled with water ($n = 1.333$). Within this aquarium, a fish observes the room. It is located close to the window and at equal distance from the room's side walls. Under which angle α does the fish see the whole room? Neglect the influence of the aquarium's window.



a) 0° b) 78° c) 90° d) 97° e) 111° f) 180°

Question 15

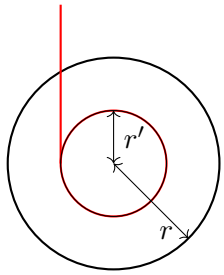
This sketch shows a container filled with water. The upper left-hand side of the container is open to the environment (standard temperature and pressure conditions). The upper right-hand side of the container is closed and the volume above the water surface is under vacuum. At equilibrium, what is the value of L? Neglect any capillary forces.



a) 1 m
b) 2 m
c) 4 m
d) 10 m
e) 12 m
f) None of the above.

Question 16

We consider a yo-yo of mass m, radius r and moment of inertia $\frac{1}{2}mr^2$. The yo-yo's string is rolled up around an inner radius $r' = r/2$. The yo-yo is released, while the string's end is held firmly. What is the ratio $E_{\text{rot.}}/E_{\text{trans.}}$ of kinetic energy due to rotation and translation while the yo-yo is rolling downwards?



a) 1 b) $\sqrt{2}$ c) $\frac{1}{3}$ d) $\sqrt{\frac{3}{2}}$ e) 2

Question 17

An electron and proton are initially placed 1 m apart. They are both released simultaneously. What are their respective initial accelerations?

a) $a_{\text{e}} = 253\,\text{m}\cdot\text{s}^{-2}$, $a_{\text{p}} = 0.138\,\text{m}\cdot\text{s}^{-2}$, going apart

b) $a_{\text{e}} = 253\,\text{m}\cdot\text{s}^{-2}$, $a_{\text{p}} = 0.138\,\text{m}\cdot\text{s}^{-2}$, coming together

c) $a_{\text{e}} = a_{\text{p}} = 2.307\times10^{-28}\,\text{m}\cdot\text{s}^{-2}$, going apart

d) $a_{\text{e}} = a_{\text{p}} = 2.307\times10^{-28}\,\text{m}\cdot\text{s}^{-2}$, coming together

e) $a_{\text{e}} = 2.82\times10^{-8}\,\text{m}\cdot\text{s}^{-2}$, $a_{\text{p}} = 1.53\times10^{-11}\,\text{m}\cdot\text{s}^{-2}$, going apart

f) $a_{\text{e}} = 2.82\times10^{-8}\,\text{m}\cdot\text{s}^{-2}$, $a_{\text{p}} = 1.53\times10^{-11}\,\text{m}\cdot\text{s}^{-2}$, coming together

Question 18

What is the order of magnitude of molecules contained in a glass of water?

a) 10^{25} b) 10^{28} c) 10^{31} d) 10^{34} e) 10^{37}

Question 19

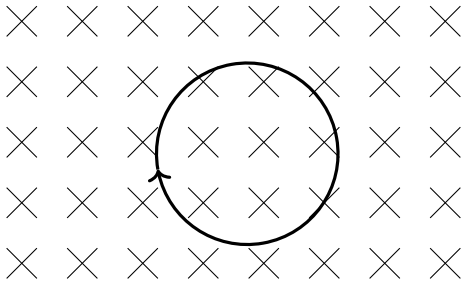
I am at rest. A car approaches me at high speed. It emits a sound which corresponds to the note A of a major scale. After the car passes me, it is keeps traveling at the same speed. I can hear the same note but reduced by an octave. What was the car's speed? The speed of sound in air is $343\,\text{m}\cdot\text{s}^{-1}$.

a) $172\,\text{km}\cdot\text{h}^{-1}$ c) $343\,\text{km}\cdot\text{h}^{-1}$ e) $617\,\text{km}\cdot\text{h}^{-1}$

b) $309\,\text{km}\cdot\text{h}^{-1}$ d) $412\,\text{km}\cdot\text{h}^{-1}$

Question 20

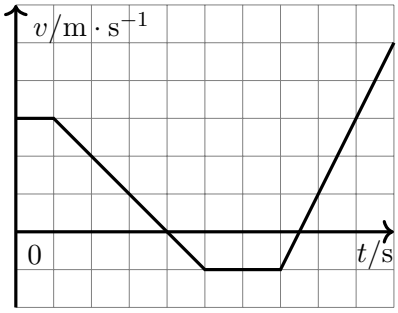
Which particle would follow this trajectory in an homogenous magnetic field?



a) A proton.
b) An antimuon.
c) An electron.
d) A photon.
e) A neutron.

Question 21

This diagramm shows a car's speed as a function of time. Which statement is false?



a) The position of the car at time $t = 5\,\mathrm{s}$ is the same as at time $t = 9\,\mathrm{s}$.

b) The average speed between $t = 0\,\mathrm{s}$ and $t = 6\,\mathrm{s}$ is $1\,\mathrm{m} \cdot \mathrm{s}^{-1}$.

c) The distance between the start and end positions is $11\,\mathrm{m}$

d) The car is at rest between $t = 5\,\mathrm{s}$ und $t = 7\,\mathrm{s}$

e) At time $t = 2\,\mathrm{s}$, the car is braking.

Question 22

The nuclear power plant at Mühleberg was decommissioned on December 20th of 2019. Before that, it produced an annual electric output of 3 TWh. The cooling was not done with a cooling tower, but by the direct use of water from the Aar (river with a flow rate of $150\,\mathrm{m}^3 \cdot \mathrm{s}^{-1}$). The temperature of the water leaving the facility was elevated by $3\,°\mathrm{C}$. A nuclear power plant has a typical efficiency coefficient of 33%. Assuming that four fifths of the energy losses were evacuated by the Aar water, what percentage of the river's flow rate passed through the cooling system? The thermal capacity of water at normal temperature and pressure is $4185.5\,\mathrm{J} \cdot \mathrm{kg}^{-1} \cdot \mathrm{K}^{-1}$.

a) 8% b) 12% c) 25% d) 30% e) 33%

Multiple choice: solutions

	a)	b)	c)	d)	e)	f)
Question 1	□	□	■	□		
Question 2	■	□	□	□	□	□
Question 3	■	□	□	□	□	
Question 4	■	□	□	□	□	
Question 5	□	□	■	□		
Question 6	□	□	■	□	□	□
Question 7	□	□	■	□		
Question 8	□	□	□	■	□	□
Question 9	□	■	□	□	□	
Question 10	■	□	□	□	□	
Question 11	□	□	■	□		
Question 12	□	□	□	■	□	□
Question 13	□	□	□	□	□	■
Question 14	□	□	□	■	□	□
Question 15	□	□	□	■	□	□
Question 16	□	□	□	□	■	
Question 17	□	■	□	□	□	□
Question 18	■	□	□	□	□	
Question 19	□	□	□	■	□	
Question 20	□	□	■	□	□	
Question 21	□	□	□	■	□	
Question 22	□	□	□	■	□	

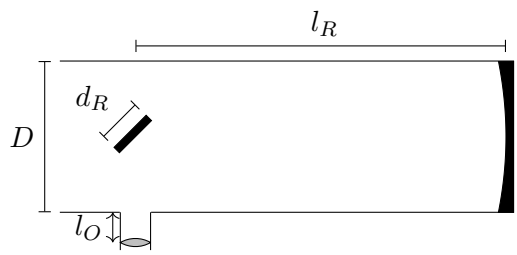
Theoretical Problems

Duration: 120 minutes
Marks: 48 points

Start each problem on a new page in order to ease the correction.

Problem 1: Reflective telescope *(16 points)*

Consider the following schematic of a mirror telescope setup, using an ocular lens. The sketch is not to scale.



The telescope has a total length of $L = 2\,\mathrm{m}$, and the diameter of the opening and the parabolic mirror is $D = 50\,\mathrm{cm}$. In the interior of the telescope, there is a planar secondary mirror with diameter $d_R = 9\,\mathrm{cm}$. The secondary mirror is placed $l_R = 1.8\,\mathrm{m}$ from the parabolic mirror, and tilted by 45°. The ocular lens has diameter $d_O = 1\,\mathrm{cm}$ and is placed at an adjustable distance l_O outside the housing, in such a way that a sharp image forms. The focal length of the parabolic mirror is $f_P = 2.1\,\mathrm{m}$, and the focal length of the ocular lens is $f_O = 3\,\mathrm{cm}$.

Part A. Geometrical optics *(8.75 points)*

i. (1 P.) Sketch the trajectory of light from a very distant star through the telescope. Use a separate piece of paper.

ii. (1.75 P.) At which distance l_O does the ocular lens need to be placed so that the rays from a very distant star are parallel after the ocular lens?

iii. (1.5 P.) Determine the magnification of the telescope.

iv. (2.5 P.) A binary star system is located at a distance of $l = 15\,\mathrm{ly}$. What is the minimum distance d between the stars so that they can be distinguished with the telescope?

v. (2 P.) At which distance l'_O does the ocular lens need to be placed to produce a sharp, 1 cm tall image of the Sun ($D_S \approx 1.4 \times 10^9\,\mathrm{m}$) on a CCD sensor behind the ocular lens?

Part B. Wave optics *(7.25 points)*

The electromagnetic spectrum of a star contains a lot of information about the star, ranging from its relative velocity, atomic composition, to its magnetic field strength. For this reason, telescopes such as this one are equipped with a spectrometer.

i. (1 P.) Sketch and label a spectrometer that could be used for taking such measurements. For this, use a reflective diffraction grating with $n = 1000$ lines per mm.

ii. (1.25 P.) At which angle can the first maximum for the red H-α line (656.281 nm) be observed? For simplicity, assume that the light falls perpendicularly onto the grating.

iii. (2.5 P.) The magnetic field of a star can manifest itself in a splitting of these spectral lines. What is the minimum measurable absolute separation of the H-α line?

iv. (2.5 P.) In this case, at which minimum distance does a photodetector need to be placed if its resolution is limited to 0.6 mm?
Hint:

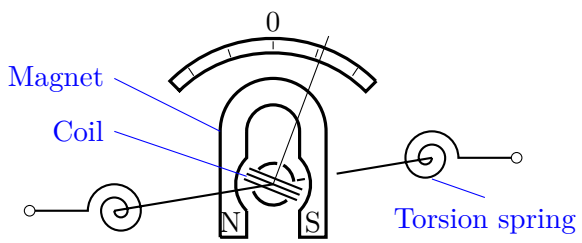
$$\frac{\mathrm{d}}{\mathrm{d}x} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

Problem 2: Electrical measures *(16 points)*

In this exercise, we will study various aspect of electrical measure instruments.

Part A. Galvanometer *(5 points)*

The figure below represents a moving coil galvanometer, also called magneto-electrical galvanometer or D'Arsonval-Weston galvanometer.



i. (0.5 P.) Was does a galvanometer measure?

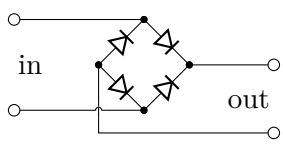
ii. (2 P.) Explain how the galvanometer described above works.

iii. (1.5 P.) Why does the magnet's central part have a circular shape? Think about the shape of the magnetic field.

iv. (1 P.) Why cannot we use this galvanometer in an AC circuit?

Part B. Diode bridge *(5 points)*

To tackle this problem, we can insert the following diode bridge in the circuit:



We assume that the diodes are ideal and have a bias voltage of zero.

i. (1 P.) Sketch or describe the current flowing through a diode as a function of the voltage across it.

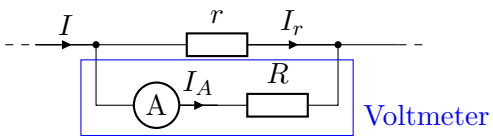
We apply an oscillating input voltage $U_{\text{in}}(t) = U_0 \sin(\omega t)$ to the diode bridge.

ii. (1.5 P.) Explain how the electrical current flows through the diode bridge at minimum and maximum values of input voltage U_{in}. Sketch the output voltage $U_{\text{out}}(t)$.

iii. (2.5 P.) What is the use of the diode bridge? Compute the time-averaged value $\bar{U}_{\text{out}}$ of the output voltage, then provide a numerical value by taking $U_0 = 5.0\,\text{V}$.

Part C. Voltmeter *(4 points)*

One way to build a voltmeter is to connect an ammeter (assumed without internal resistance) and a resistor R in series. The diagram above shows such an assembly in the case where one wants to measure the voltage accross an element with resistance r.



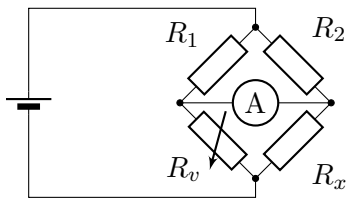
i. (0.25 P.) Compute the voltage across the element as a function of the resistance R and of the electrical current I_A measured by the ammeter.

ii. (1.75 P.) Provided the electrical current which enters the assembly is I, compute the electrical current I_r flowing through the element as a function of I, r and R.

iii. (2 P.) We wish to minimize the perturbation induced by the voltmeter. How should we choose R and what impact will it have on the ammeter?

Part D. Wheatstone bridge *(2 points)*

A way of measuring resistances is to use an assembly called a Wheatstone bridge:

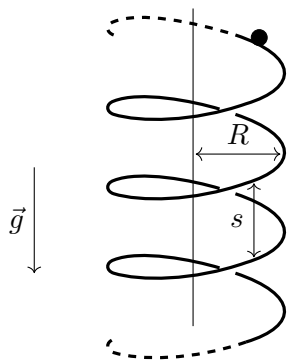


To find the unknown resistance R_x, we vary R_v such that the ammeter indicates zero.

i. (2 P.) In this condition, compute R_x as a function of R_1, R_2 and R_v. Explain your reasoning.

Problem 3: Helical slope *(16 points)*

Let's consider a helical ramp. The helix's axis is vertical, its radius R (the horizontal distance from each point of the ramp to the axis) is constant. The ramp's slope is also constant and such that the vertical distance between two coils (distance which is called the helix's "pitch") is s.



We study the motion of a marble of mass m that rolls on the ramp. The marble's position $l(t)$ on the helix is described by the distance it travelled along the ramp from its initial position.

Part A. A point object on a line *(7 points)*

First, let's consider that the ramp is analog to a line along which the marble moves without friction and without leaving the ramp.

i. (1 P.) What is the length L of one helix's turn, that is, the distance the marble travelled when it crosses the vertical of its initial position for the first time after being let go along the ramp?

ii. (1 P.) What is the angle α between the ramp and the horizontal plane?

iii. (1 P.) Draw the applied forces acting on the marble in the referential of your choice.

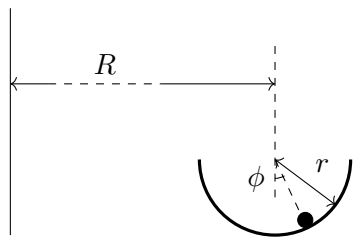
iv. (1.5 P.) Compute the marble's acceleration $a(t)$ tangent to the ramp as a function of time.

v. (0.5 P.) We let the marble roll along the ramp with an initial velocity v_0 (tangent to the ramp). Compute the marble's position $l(t)$ as a function of time.

vi. (2 P.) If the initial velocity's direction is upward the ramp, after what time τ will the marble cross its initial position again? Find a numerical value for τ using $R = s = 20\,\mathrm{cm}$ and $v_0 = 1\,\mathrm{m\cdot s^{-1}}$.

Part B. Like a slide *(9 points)*

We consider now that the ramp's cross-section is a half-circle of radius r, the two rims being at same height. The marble is still considered a point object with frictionless motion. The position of the marble inside the ramp is determined by the angle $\phi(t)$ (taken in the vertical plane containing the helix's axis) and the distance $l(t)$ from its initial position, measured along the bottom of the half-pipe (where $\phi = 0$).



i. (5 P.) Find an equation that links the variables $\phi(t)$, R, r, s, $l(t)$ and L if the marble's initial conditions are $v_0 = 0$ and $\phi_0 = 0$. No other variable than these six ones should appear in this equation. **You are not asked to solve this equation.**

ii. (1 P.) Will the marble jump off the ramp?

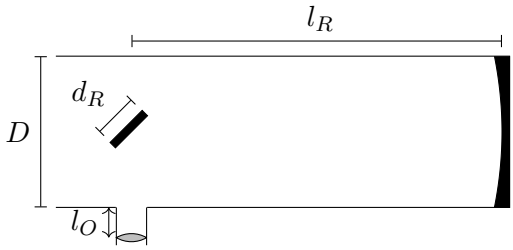
iii. (2 P.) How is the equation simplified if we assume $R \gg r$?

iv. (1 P.) Provide the numerical value of $\phi(t)$ when the marble completed 5 turns (with $R = 10\,\mathrm{m}$, $r = 2\,\mathrm{cm}$ and $s = 2\,\mathrm{m}$).

Theoretical Problems: solutions

Problem 1: Reflective telescope *(16 points)*

Consider the following schematic of a mirror telescope setup, using an ocular lens. The sketch is not to scale.



The telescope has a total length of $L = 2\,\mathrm{m}$, and the diameter of the opening and the parabolic mirror is $D = 50\,\mathrm{cm}$. In the interior of the telescope, there is a planar secondary mirror with diameter $d_R = 9\,\mathrm{cm}$. The secondary mirror is placed $l_R = 1.8\,\mathrm{m}$ from the parabolic mirror, and tilted by $45°$. The ocular lens has diameter $d_O = 1\,\mathrm{cm}$ and is placed at an adjustable distance l_O outside the housing, in such a way that a sharp image forms. The focal length of the parabolic mirror is $f_P = 2.1\,\mathrm{m}$, and the focal length of the ocular lens is $f_O = 3\,\mathrm{cm}$.

Part A. Geometrical optics *(8.75 points)*

i. (1 P.) Sketch the trajectory of light from a very distant star through the telescope. Use a separate piece of paper.

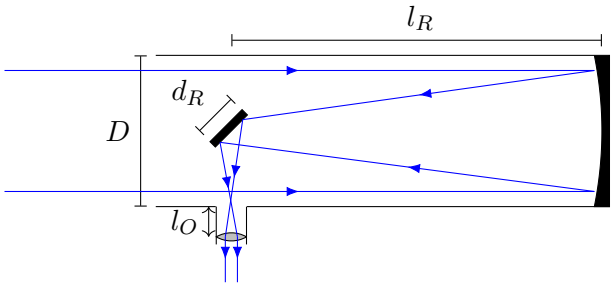
[0.25 P.] Quasi Parallele Strahlen vom Stern vor dem Spiegel

[0.25 P.] Konvergierende Strahlen nach Parabolspiegel

[0.25 P.] Strahlen kreuzen sich nach Planarspiegel, vor Okular

[0.25 P.] Strahlen quasi parallel nach Okular

Kein Abzug, falls Strahlen im schematischen Aufbau skizziert, dies so klar deklariert und Aufgabenblatt klar mit diesen Lösungen abgegeben wird. Der Skizze muss nicht nachgerennt werden. Strahlen können mit Lineal oder freihand gezogen werden, wichtig ist, dass die Skizze soweit klar ist.



ii. (1.75 P.) At which distance l_O does the ocular lens need to be placed so that the rays from a very distant star are parallel after the ocular lens?

[0.75 P.] Damit müssen die Brennpunkte aufeinander liegen.

[0.25 P.] Daher:

$$f_P + f_O = l_R + \frac{1}{2}D + l_O$$

[0.25 P.] Somit:

$$l_O = f_P + f_O - l_R - \frac{1}{2}D$$

[0.5 P.] Numerisch:

$$l_O = 8\,\mathrm{cm}$$

Wenn jemand das Ganze versteht, kann das Endresultat direkt im Kopf berechnet werden. Dies gibt auch volle Punktzahl.

iii. (1.5 P.) Determine the magnification of the telescope.

[1 P.] Die (Winkel-)Vergrösserung für ein Teleskop (afokal) ist definiert als:

$$M = \frac{f_P}{f_O}$$

[0.5 P.] Numerisch:

$$M = 70$$

Ein negatives Vorzeichen wird erlaubt, es hängt vom gewählten Koordinatensystem nach dem Sekundärspiegel ab.

iv. (2.5 P.) A binary star system is located at a distance of $l = 15\,\mathrm{ly}$. What is the minimum distance d between the stars so that they can be distinguished with the telescope?

[0.75 P.] Winkelauflösung

$$\theta \approx 1.22\frac{\lambda}{D}$$

[0.75 P.] Verwende $\tan\theta \approx \theta$ for $\theta \ll 1$:

$$\frac{d}{l} \approx 1.22\frac{\lambda}{D}$$
$$d \approx 1.22\frac{\lambda l}{D}$$

[0.5 P.] Für λ muss eine sinnvolle Wellenlänge angenommen werden: $\lambda \approx 500\,\mathrm{nm}$

[0.5 P.] Damit der numerische Wert: (genauer Wert hängt vom angenommenen λ ab) (keine bevorzügte Einheit)

$$d \approx 1.2\,\mathrm{au} \approx 1.7 \times 10^8\,\mathrm{km}$$

v. (2 P.) At which distance l'_O does the ocular lens need to be placed to produce a sharp, $1\,\mathrm{cm}$ tall image of the Sun ($D_S \approx 1.4 \times 10^9\,\mathrm{m}$) on a CCD sensor behind the ocular lens?

[0.5 P.] Für die Distanz l' zwischen reelem Bild des Parabolspiegels und Okular muss gelten:

$$B' = \frac{f_O}{f_O - l'} B_1 = \frac{f_O}{f_O - l'} \frac{f_P}{f_P - l_0} D_S$$

[0.5 P.] und somit:

$$\begin{aligned} f_O - l' &= f_O \frac{f_P}{f_P - l_0} \frac{D_S}{B'} \\ l' &= f_O \left(1 + \frac{f_P}{l_0 - f_P} \frac{D_S}{B'}\right) \end{aligned}$$

[0.5 P.] und daraus:

$$\begin{aligned} l_R + \frac{1}{2}D + l'_O &= l' + l_1 \\ l'_O &= f_O \left(1 + \frac{f_P}{l_0 - f_P} \frac{D_S}{B'}\right) + f_P \frac{l_0}{l_0 - f_P} - l_R - \frac{1}{2}D \end{aligned}$$

[0.5 P.] Durch einsetzen der Zahlen ($l_0 = 1\,\mathrm{au}$):

$$l'_O = 14\,\mathrm{cm}$$

Part B. Wave optics *(7.25 points)*

The electromagnetic spectrum of a star contains a lot of information about the star, ranging from its relative velocity, atomic composition, to its magnetic field strength. For this reason, telescopes such as this one are equipped with a spectrometer.

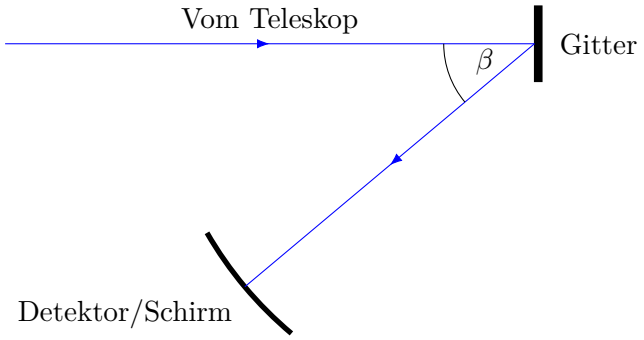
i. (1 P.) Sketch and label a spectrometer that could be used for taking such measurements. For this, use a reflective diffraction grating with $n = 1000$ lines per mm.

[0.25 P.] Gitter klar und richtig

[0.25 P.] Strahlweg klar und richtig

[0.25 P.] Schirm/Detektor klar und richtig

[0.25 P.] Sinnvoll beschriftet



ii. (1.25 P.) At which angle can the first maximum for the red H-α line (656.281 nm) be observed? For simplicity, assume that the light falls perpendicularly onto the grating.

[0.75 P.] Konstruktive Interferenz für

$$\sin\beta = \lambda n$$
$$\beta = \arcsin(\lambda n)$$

[0.5 P.] Und der numerische Wert:

$$\beta = 41.017°$$

iii. (2.5 P.) The magnetic field of a star can manifest itself in a splitting of these spectral lines. What is the minimum measurable absolute separation of the H-α line?

[0.75 P.] Die relative Auflösung ergibt sich aus:

$$\frac{\delta\lambda}{\lambda} = \frac{1}{N}$$

wobei N die Anzahl bestrahlte Linien sind.

[0.5 P.] Bei parallel ausfallenden Strahlen aus dem Okular werden insgesamt

$$N = d_O n$$

Linien bestrahlt. Das ist zwar nur eine Näherung aber gibt doch die richtige Grössenordnung.

[0.25 P.] Damit ist

$$\frac{\delta\lambda}{\lambda} = \frac{1}{d_O n}$$

[0.5 P.] Und es folgt die absolute Auflösung und die H-α

$$\delta\lambda = \frac{\lambda}{d_O n}$$

[0.5 P.] und numerisch:

$$\delta\lambda = 0.07\,\mathrm{nm}$$

iv. (2.5 P.) In this case, at which minimum distance does a photodetector need to be placed if its resolution is limited to $0.6\,\mathrm{mm}$**?**
Hint:

$$\frac{\mathbf{d}}{\mathbf{d}x}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

[0.5 P.] Die oben berechnete Auflösung $\delta\lambda$ entspricht einer Winkelauflösung von

$$\delta\beta = \frac{\partial\beta}{\partial\lambda}\delta\lambda$$

[0.5 P.] Damit

$$\begin{aligned}\delta\beta &= \delta\lambda\frac{\partial}{\partial\lambda}\arcsin(\lambda n)\\ &= \delta\lambda\frac{n}{\sqrt{1-(n\lambda)^2}}\end{aligned}$$

Dabei wird der Zusammenhang aus B.ii verwendet.

[0.5 P.] Bei bekannter Positionsauflösung entspricht dies einem Radius von

$$R = \frac{\delta x}{\delta\beta}$$

[0.5 P.]

$$\begin{aligned}R &= \frac{\sqrt{1-(n\lambda)^2}}{n}\frac{\delta x}{\delta\lambda}\\ &= \sqrt{1-(n\lambda)^2}\frac{d_o}{\lambda}\delta x\end{aligned}$$

[0.5 P.] und numerisch:

$$R = 6.9\,\mathrm{m}$$

[-1 P.] Die Näherung $\sin x \approx x$ ist hier nicht zulässig, da wir deutlich weg von einer Nullstelle sind. Dieser Fehler vereinfacht die nachfolgenden Rechnungen, sodass 1P anstatt nur 0.5P abgezogen werden.

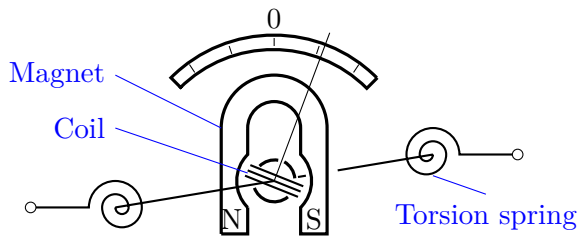
Allenfalls kann man auch zwei konkrete Winkel für die Berechnung verwendet und ein korrektes Resultat erhalten werden. Dies (wenn komplet richtig) wird mit der vollen Punktzahl bewertet.

Problem 2: Electrical measures *(16 points)*

In this exercise, we will study various aspect of electrical measure instruments.

Part A. Galvanometer *(5 points)*

The figure below represents a moving coil galvanometer, also called magneto-electrical galvanometer or D'Arsonval-Weston galvanometer.



i. (0.5 P.) Was does a galvanometer measure?

[0.5 P.] A galvanometer is an (analogic) ammeter, i.e. it measures currents.

ii. (2 P.) Explain how the galvanometer described above works.

[1 P.] When current flows through the coil, the magnet applies Lorentz/Laplace force on the coil, thereby creating a couple and thus a torque on the coil/pointer part.

Alternative description: when current flows through the coil, it creates a magnetic field (Oersted's law) (0.5 point). This magnetic field interacts with the one of the permanent magnet, creating a torque on the coil/pointer part (0.5 point).
[0.5 P.] The torsion springs counterbalance this torque.

[0.5 P.] At equilibrium (for a constant current), the coil/pointer part has a fixed deviation. The pointer indicates the value of the current on the (pre-calibrated) scale.

iii. (1.5 P.) Why does the magnet's central part have a circular shape? Think about the shape of the magnetic field.

[0.5 P.] That way, the magnetic field is radially symmetric, so the torque from the magnetic field (linear in the current from Laplace force) is independent of the angular deviation.

[0.5 P.] The torque of the torsion springs is linear in the angular deviation.

[0.5 P.] Therefore with a circular inter-magnet space, the deviation is linear in the current, which simplifies calibration.

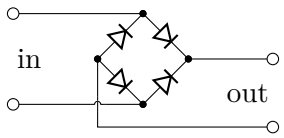
We can give 0.5 point for a correct drawing of the magnetic field (but the total of the question should be kept at max. 1.5).

iv. (1 P.) Why cannot we use this galvanometer in an AC circuit?

[1 P.] The current switches back and forth, the induced magnetic field too. Therefore the pointer stays at 0 (as such an instrument is not sensitive enough to rapid changes in the current).

Part B. Diode bridge *(5 points)*

To tackle this problem, we can insert the following diode bridge in the circuit:

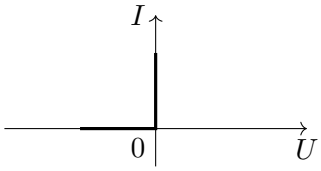


We assume that the diodes are ideal and have a bias voltage of zero.

i. (1 P.) Sketch or describe the current flowing through a diode as a function of the voltage across it.

For a sketch:
[0.5 P.] Axes labelled.

[0.5 P.] 0 for negative voltage, infinite above.



For an explanation: 0.5 point for having zero current (or equivalently, infinite resistance) for negative tension, 0.5 point for having infinite current (or equivalently, zero resistance) for positive tension.
Points can be cumulated between sketch and explanation, but only up to the one full point.
We apply an oscillating input voltage $U_{\mathbf{in}}(t) = U_0 \sin(\omega t)$ to the diode bridge.

ii. (1.5 P.) Explain how the electrical current flows through the diode bridge at minimum and maximum values of input voltage $U_{\mathbf{in}}$. Sketch the output voltage $U_{\mathbf{out}}(t)$.

Let's fix the input tension to be the voltage of the upper node minus the voltage of the lower one.
[0.25 P.] At maximum input tension, the current from the upper input node can only flow through the upper right diode to the upper output node. The current in the lower input node can only flow into it, from the lower output node via the lower left diode. No current flows through the two other diodes.

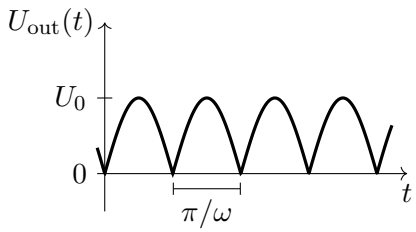
[0.25 P.] At minimum input tension (negative), the current from the lower input node can only flow through the lower right diode to the upper output node. The current in the upper input node can only flow into it, from the lower output node via the upper left diode. No current flows through the two other diodes.

[0.5 P.] Axes labelled; ω, U_0 and 0 voltage present.

[0.5 P.] The plot should more or less look like:

$$U_0 \left|\sin(\omega t)\right|$$

(The next subtask gives the rationale behind this equation.)



iii. (2.5 P.) What is the use of the diode bridge? Compute the time-averaged value $\bar{U}_{\text{out}}$ of the output voltage, then provide a numerical value by taking $U_0 = 5.0\,\text{V}$.

[0.5 P.] From the two first subtasks, the output tension will always have the same norm as the input. But the upper node will always get the higher voltage. This is to say that the output tension is (assuming U_0 positive):

$$U_0\,|\sin(\omega t)|$$

[0.5 P.] The diode bridge is useful because the output tension stays positive. The AC is therefore turned into (non-constant) DC. This element acts as a rectifier.

[0.5 P.] This mean value corresponds to $U_{\text{out}}(t)$ averaged over $\frac{\pi}{\omega}$ (that is, a period of $|\sin(\omega t)|$ resp. half a period of $\sin(\omega t)$).

[0.5 P.]

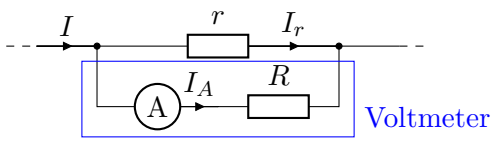
$$\begin{aligned}\bar{U}_{\text{out}} &= U_0\frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}}\sin(\omega t)\,\mathrm{d}t\\ &= U_0\frac{\omega}{\pi}\left[-\frac{\cos(\omega t)}{\omega}\right|_0^{\frac{\pi}{\omega}}\\ &= U_0\frac{1}{\pi}\left(-(-1)+1\right)\\ &= \frac{2}{\pi}U_0\end{aligned}$$

[0.5 P.] Numerically:

$$\bar{U}_{\text{out}} \approx 3.2\,\text{V}$$

Part C. Voltmeter *(4 points)*

One way to build a voltmeter is to connect an ammeter (assumed without internal resistance) and a resistor R in series. The diagram above shows such an assembly in the case where one wants to measure the voltage accross an element with resistance r.



i. (0.25 P.) Compute the voltage across the element as a function of the resistance R and of the electrical current I_A measured by the ammeter.

[0.25 P.] Ohm's law.

$$U = RI_A$$

ii. (1.75 P.) Provided the electrical current which enters the assembly is I, compute the electrical current I_r flowing through the element as a function of I, r and R.

[0.5 P.] Ohm's law on the element:

$$U = rI_r$$

[0.5 P.] Kirchhoff's law on currents:

$$I = I_A + I_r$$

[0.75 P.] Thus:

$$\begin{aligned} I_r &= I - I_A \\ &= I - \frac{U}{R} \\ &= I - \frac{r}{R} I_r \end{aligned}$$

Reordering:

$$I_r = \frac{1}{1 + \frac{r}{R}} I$$

(Any correct algebraic transformation leading to the correct answer gives the full 0.75 point, no need to follow the exact same steps.)

iii. (2 P.) We wish to minimize the perturbation induced by the voltmeter. How should we choose R and what impact will it have on the ammeter?

[0.25 P.] We want I_r to be as close as possible to I.

[0.75 P.] Therefore we have to choose $R \gg r$.

[0.25 P.] But: (Ohm's law at fixed U)

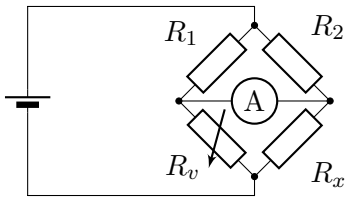
$$I_A \propto \frac{1}{R}$$

That is, I_A will get very small as R grows relatively.

[0.75 P.] So we need a highly sensitive ammeter.

Part D. Wheatstone bridge *(2 points)*

A way of measuring resistances is to use an assembly called a Wheatstone bridge:



To find the unknown resistance R_x, we vary R_v such that the ammeter indicates zero.

i. (2 P.) In this condition, compute R_x as a function of R_1, R_2 and R_v. Explain your reasoning.

[0.5 P.] The ammeter indicates zero when there is no current flowing through it, which means that there is no tension between its electrodes.

[0.5 P.] Therefore we have:

$$I_1 R_1 = I_2 R_2$$

[0.5 P.] And:

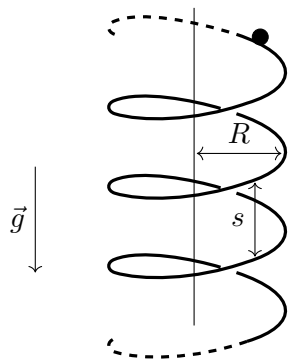
$$I_1 R_v = I_2 R_x$$

[0.5 P.] Finally:

$$\begin{aligned} R_x &= \frac{I_1}{I_2} R_v \\ &= \frac{R_2}{R_1} R_v \end{aligned}$$

Problem 3: Helical slope *(16 points)*

Let's consider a helical ramp. The helix's axis is vertical, its radius R (the horizontal distance from each point of the ramp to the axis) is constant. The ramp's slope is also constant and such that the vertical distance between two coils (distance which is called the helix's "pitch") is s.



We study the motion of a marble of mass m that rolls on the ramp. The marble's position $l(t)$ on the helix is described by the distance it travelled along the ramp from its initial position.

Part A. A point object on a line *(7 points)*

First, let's consider that the ramp is analog to a line along which the marble moves without friction and without leaving the ramp.

i. (1 P.) What is the length L of one helix's turn, that is, the distance the marble travelled when it crosses the vertical of its initial position for the first time after being let go along the ramp?

[1 P.] One can 'unroll' the helix and use the Pythagorean theorem:

$$L = \sqrt{(2\pi R)^2 + s^2}$$

ii. (1 P.) What is the angle α between the ramp and the horizontal plane?

[0.5 P.] Again by unrolling:

$$\tan(\alpha) = \frac{s}{2\pi R}$$

[0.5 P.] And thus:

$$\alpha = \arctan\left(\frac{s}{2\pi R}\right)$$

iii. (1 P.) Draw the applied forces acting on the marble in the referential of your choice.

The forces are the gravity, the normal force from the ramp. The normal force is not vertical, there is a component in the radial direction towards the center. Alternatively the students can also break the normal force into its vertical and radial component.

[0.25 P.] Referential (origin and axes labelled)

[0.25 P.] Gravity

[0.25 P.] Normal force (or both its projections)

[0.25 P.] Correct drawing of the forces in the referential

iv. (1.5 P.) Compute the marble's acceleration $a(t)$ tangent to the ramp as a function of time.

[0.5 P.] The tangential acceleration is constant.

[0.5 P.] And, as there is no friction, it is given by projecting $\vec{g}$ onto the trajectory.

$$a = g\sin(\alpha)$$

[0.5 P.] Which can be rewritten (the full half-point is also given if this rewriting appears only later)

$$a = \frac{gs}{L}$$

v. (0.5 P.) We let the marble roll along the ramp with an initial velocity v_0 (tangent to the ramp). Compute the marble's position $l(t)$ as a function of time.

[0.5 P.] The (scalar) acceleration is constant, so l behaves like in a uniformly accelerated movement; general equation:

$$r(t) = r_0 + v_0 t + \frac{1}{2}at^2$$

So we have, from the way l is defined:

$$l(t) = v_0 t + \frac{1}{2}at^2$$

vi. (2 P.) If the initial velocity's direction is upward the ramp, after what time τ will the marble cross its initial position again? Find a numerical value for τ using $R = s = 20\,\mathrm{cm}$ and $v_0 = 1\,\mathrm{m}\cdot\mathrm{s}^{-1}$.

[0.5 P.] We want $l(\tau) = 0$.

[1 P.]

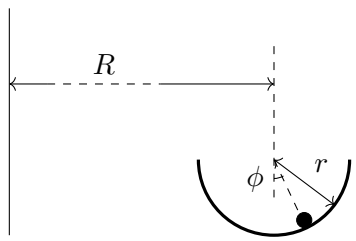
$$\begin{aligned}
0 &= v_0 + \frac{1}{2}a\tau \\
\tau &= \frac{2v_0}{a} \\
&= \frac{2v_0}{gs}\sqrt{(2\pi R)^2 + s^2} \\
&= \frac{2v_0}{g}\sqrt{\left(2\pi\frac{R}{s}\right)^2 + 1}
\end{aligned}$$

[0.5 P.] Numerically:

$$\tau \approx 1.3\,\mathrm{s}$$

Part B. Like a slide *(9 points)*

We consider now that the ramp's cross-section is a half-circle of radius r, the two rims being at same height. The marble is still considered a point object with frictionless motion. The position of the marble inside the ramp is determined by the angle $\phi(t)$ (taken in the vertical plane containing the helix's axis) and the distance $l(t)$ from its initial position, measured along the bottom of the half-pipe (where $\phi = 0$).



i. (5 P.) Find an equation that links the variables $\phi(t)$, R, r, s, $l(t)$ and L if the marble's initial conditions are $v_0 = 0$ and $\phi_0 = 0$. No other variable than these six ones should appear in this equation. You are not asked to solve this equation.

[1 P.] One can consider that the radial velocity is always much smaller that the tangential one (and changes very little), therefore does not contribute an acceleration. (This does not need to be explicitly written, the point can be given if the calculations are correct or clearly use this assumption.)

[1 P.] Forces are the normal force $\vec{N}$ and gravity $m\vec{g}$

$$N\cos(\phi) - mg = 0$$
$$N\sin(\phi) = ma_c = m\frac{v^2}{R + r\sin(\phi)}$$

[1 P.] Normal condition (Newton): (the developped form is not requested for the full point)

$$\frac{v^2}{(R + r\sin(\phi))} = g\tan(\phi)$$

[1 P.] Energy conservation:

$$\frac{1}{2}mv^2 = mg\left(s\frac{l}{L} - r\left(1 - \cos(\phi)\right)\right)$$

[1 P.] Merging both equations via v^2 and cancelling the g (not requested for the full point, as it is not necessarily clear whether g is a variable or a constant...) (the exact form of the equation is not relevant, it should only be equivalent to this one):

$$\tan(\phi)\left(R + r\sin(\phi)\right) = 2\left(s\frac{l}{L} - r\left(1 - \cos(\phi)\right)\right)$$

ii. (1 P.) Will the marble jump off the ramp?

[1 P.] Not in a finite distance, as from the previous task $\tan(\phi)$ will remain finite for a finite distance.

Note: the following precisions are valid but not required for the full point:

- The marble will come so close to the edge of the ramp that in reality even a small perturbation could make it jump off.
- The conclusion is only valid for $R > r$.

Note: more complete mathematical treatment, not asked for: the equation can be simplified using the half-angle formula, with $x = \tan\left(\frac{\theta}{2}\right)$:

$$\left(4\frac{r}{R} - 2\frac{sl}{RL}\right)x^4 - 2x^3 - 2x + 2\frac{sl}{RL} = 0$$

For $0 \leq r < R$, the branch of $x(l)$ starting at 0 converges towards 1 for growing l. For $r > R$ there is a gap of forbidden values for l indicating that the marble will jump off. For $r = R$ there is an instability leading to possible jump off at each $l \geq \frac{R}{s}L$.

iii. (2 P.) How is the equation simplified if we assume $R \gg r$?

[1 P.] Dividing by R: (if this was already done above, the full point is also given)

$$\tan(\phi)\left(1 + \frac{r}{R}\sin(\phi)\right) = \frac{2sl}{LR} - 2\frac{r}{R}\left(1 - \cos(\phi)\right)$$

[1 P.] $\sin(\phi)$ and $\cos(\phi)$ are bounded, so:

$$\tan(\phi) \approx \frac{2sl}{LR}$$

iv. (1 P.) Provide the numerical value of $\phi(t)$ when the marble completed 5 turns (with $R = 10\,\mathrm{m}$, $r = 2\,\mathrm{cm}$ and $s = 2\,\mathrm{m}$).

[1 P.] We use the simplified formula ($R \gg r$) and $\frac{l}{L} = 5$ (either of the radian or degree numerical value gives the full point, but only if identified as such, e.g. not 1.1°):

$$\begin{aligned}\phi &\approx \arctan\left(\frac{2sl}{LR}\right)\\ &\approx 1.1\\ &\approx 63^\circ\end{aligned}$$