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Physics Olympiad

Second Round

online, 13 January 2021

Theory	**:**	**21 MC questions**
Duration	:	60 minutes
Total	:	21 points (21×1)
Authorized material	:	Calculator without database Writing and drawing material One A4 double-sided handwritten page of notes

Good luck!

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Natural constants

Hyperfine transition frequency of caesium	$\Delta\nu_{\mathrm{Cs}}$	$=$	$9\,192\,631\,770$	$\mathrm{s^{-1}}$
Speed of light in vacuum	c	$=$	$299\,792\,458$	$\mathrm{m \cdot s^{-1}}$
Planck constant	h	$=$	$6.626\,070\,15 \times 10^{-34}$	$\mathrm{kg \cdot m^2 \cdot s^{-1}}$
Elementary charge	e	$=$	$1.602\,176\,634 \times 10^{-19}$	$\mathrm{A \cdot s}$
Boltzmann constant	k_{B}	$=$	$1.380\,649 \times 10^{-23}$	$\mathrm{kg \cdot m^2 \cdot K^{-2} \cdot s^{-2}}$
Avogadro constant	N_{A}	$=$	$6.022\,140\,76 \times 10^{23}$	$\mathrm{mol^{-1}}$
Luminous efficacy	K_{cd}	$=$	683	$\mathrm{cd \cdot kg \cdot m^2 \cdot s^3}$
Magnetic constant	μ_0	$=$	$4\pi \times 10^{-7}$	$\mathrm{kg \cdot m \cdot A^{-2} \cdot s^{-2}}$
Electric constant	ε_0	$\approx$	$8.854\,187\,82 \times 10^{-12}$	$\mathrm{A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}}$
Gas constant	R	$\approx$	$8.314\,462\,618$	$\mathrm{kg \cdot m^2 \cdot K^{-1} \cdot mol^{-1} \cdot s^{-2}}$
Stefan-Boltzmann constant	σ	$\approx$	$5.670\,374\,419 \times 10^{-8}$	$\mathrm{kg \cdot K^{-4} \cdot s^{-3}}$
Gravitational constant	G	$=$	$6.674\,30(15) \times 10^{-11}$	$\mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$
Electron mass	m_{e}	$=$	$9.109\,383\,701\,5(28) \times 10^{-31}$	kg
Neutron mass	m_{n}	$=$	$1.674\,927\,498\,04(95) \times 10^{-27}$	kg
Proton mass	m_{p}	$=$	$1.672\,621\,923\,69(51) \times 10^{-27}$	kg
Standard acceleration of gravity	g_{n}	$=$	$9.806\,65$	$\mathrm{m \cdot s^{-2}}$

Multiple Choice: answer sheet

Duration: 60 minutes
Marks: 21 points (1 point for each correct answer)
Indicate your answers in the corresponding boxes on this page or write them down on a separate sheet of paper. In the latter case make clear to which question the answers belong and write down your name on the sheet of paper as well.

- Multiple-Choice **(MC)** questions have several statements, of which **exactly one** is correct. If you mark exactly the right answer on the answer sheet, you get one point, otherwise zero.

- Multiple-True-False questions **(MTF)** have multiple statements and you must decide **for each statement** whether it is true or false. If you have classified all statements correctly, you get one point. If you have misclassified one statement and all others correctly, you get 0.5 points. If you have more than one misclassified statement, you do not get any points.

Name:	**First name:**	**Total:**

	A)	B)	C)	D)	E)	F)
Question 1	☐	☐	☐	☐		
Question 2	☐	☐	☐	☐		
Question 3	☐	☐	☐	☐	☐	☐
Question 4	☐	☐	☐	☐	☐	
Question 5	☐	☐	☐	☐		
Question 6	☐	☐	☐	☐		
Question 7	☐	☐	☐	☐		
Question 8	☐	☐	☐	☐		
Question 9	☐	☐	☐	☐	☐	
Question 10	☐	☐	☐	☐		
Question 11	☐	☐	☐	☐		
Question 12	☐	☐	☐	☐	☐	
Question 13	☐	☐	☐	☐	☐	☐
Question 14	☐	☐	☐	☐		
Question 15	☐	☐	☐	☐		
Question 16	☐	☐	☐	☐	☐	
Question 17	☐	☐	☐	☐	☐	
Question 18	☐	☐	☐	☐		
Question 19	☐	☐	☐	☐	☐	
Question 20	☐	☐	☐	☐	☐	
Question 21	☐	☐	☐	☐		

Question 1 (MC)

How many tonnes of chocolate are consumed each year in Switzerland on average? (Swiss population: 8.57 millions)

A) 10^2 t B) 10^3 t C) 10^4 t D) 10^5 t

Question 2 (MTF)

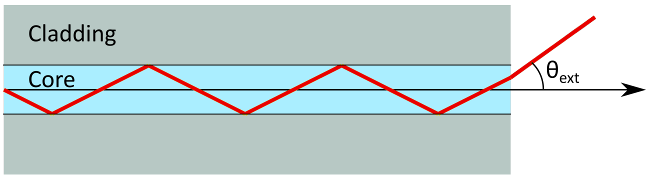
To explain experiments, "complicated" functions can be approximated for simplicity. A local linear approximation is often sufficient. Assuming $x \approx 0$ and $y \approx \frac{\pi}{2}$, which of the following statements are correct?

A) $\sin(x) \approx x$ B) $\sin(y) \approx 0$

C) $\cos(x) \approx 1$ D) $\exp(x) \approx 1 + x$

Question 3 (MC)

An optical fiber consists of several parts. The core (in the middle) is made out of glass ($n_{core} = 1.49$), and is used to transmit the light. It is surrounded by the cladding ($n_{cladding} = 1.47$), which in turn is protected by a layer of plastic. What can you tell about the angle θ_{ext} when the light exits the optical fiber?



A) $\theta_{\text{ext}} \geq \arcsin\left(\frac{n^2_{\text{core}}}{n^2_{\text{cladding}}}\right)$

B) $\theta_{\text{ext}} \geq \arcsin\left(\sqrt{n^2_{\text{core}} - n^2_{\text{cladding}}}\right)$

C) $\theta_{\text{ext}} \geq \sqrt{n^2_{\text{core}} - n^2_{\text{cladding}}}$

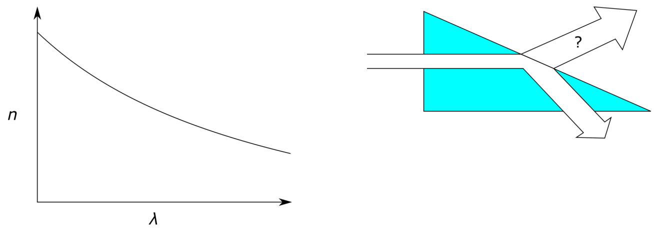
D) $\theta_{\text{ext}} \leq \arcsin\left(\frac{n^2_{\text{core}}}{n^2_{\text{cladding}}}\right)$

E) $\theta_{\text{ext}} \leq \arcsin\left(\sqrt{n^2_{\text{core}} - n^2_{\text{cladding}}}\right)$

F) $\theta_{\text{ext}} \leq \sqrt{n^2_{\text{core}} - n^2_{\text{cladding}}}$

Question 4 (MC)

On the left in the diagram below, the refractive index for different wavelengths of visible light in glass is shown. A ray of white light propagates through glass, reaches its surface with air (as shown on the right in the figure below), and is for the most part totally reflected. What is the color of the part of the light that leaves the glass and continues to propagate through the air?



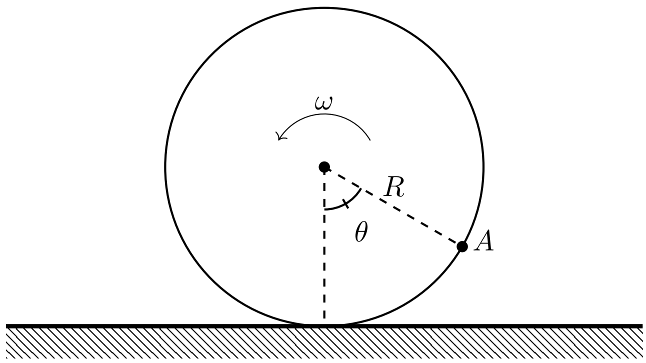
A) Red B) Blue

C) Black D) White

E) Mint green

Question 5 (MC)

A wheel of radius R is rolling without slipping with an angular velocity ω.



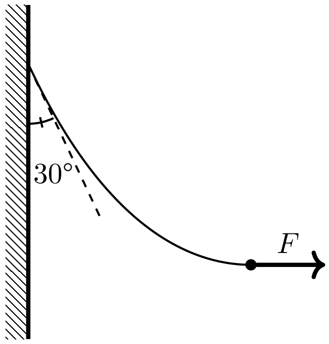
As shown in the figure, point A on the wheel makes an angle θ with respect to the vertical. What is the magnitude of its velocity with respect to the reference frame of the ground?

A) $2\omega R\,|\sin\theta|$ B) $\omega R\,|\sin\theta|$

C) $2\omega R\left|\sin\frac{\theta}{2}\right|$ D) $\omega R\left|\sin\frac{\theta}{2}\right|$

Question 6 (MC)

One end of a rope is fixed to a vertical wall and the other end pulled by a horizontal force of F. The angle made by the rope and the vertical wall is 30° as shown in the figure.



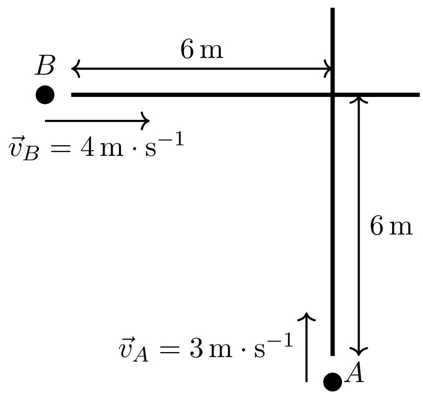
What is the weight W of the rope?

A) $W = \frac{\sqrt{3}}{2}F$ B) $W = \frac{1}{2}F$

C) $W = \frac{1}{\sqrt{3}}F$ D) $W = \sqrt{3}F$

Question 7 (MC)

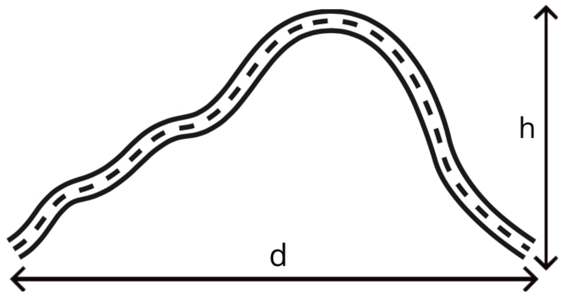
Particle A is travelling northward at a constant speed $3\,\mathrm{m \cdot s^{-1}}$, while particle B is travelling eastward at a constant speed $4\,\mathrm{m \cdot s^{-1}}$. At a certain instant, both particles are at the distance 6 m from the intersection point of their trajectories, as shown in the figure. What is the minimal distance between the particles in this motion?



A) 0.8 m B) 1 m C) 1.2 m D) 1.4 m

Question 8 (MC)

A frictionless tube lies in the vertical plane and has the shape, as shown in the sketch, so that its two endpoints are at the same height. A chain with uniform mass per unit length μ is placed into the tube from end to end and released. What is the net force acting on the chain?



A) $\mu g h$

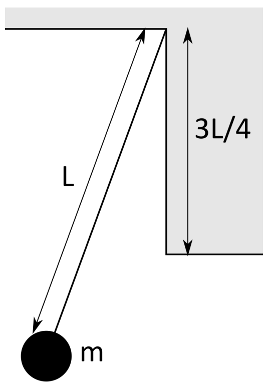
B) $\mu g d$

C) 0

D) Not enough information given.

Question 9 (MC)

A pendulum of length L and mass m is attached in a corner. What is the ratio of the time T_1 that the mass spends on the left part to the time T_2 spent in the right part?

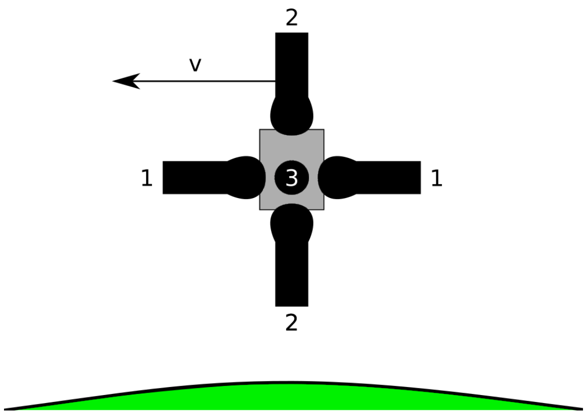


A) $\frac{T_1}{T_2} = \frac{1}{4}$ B) $\frac{T_1}{T_2} = \frac{1}{2}$ C) $\frac{T_1}{T_2} = 1$

D) $\frac{T_1}{T_2} = 2$ E) $\frac{T_1}{T_2} = \sqrt{3}$

Question 10 (MC)

The Space Force is building a satellite to shoot cannonballs to the earth from orbit. One cannon is oriented forward, one backward, one pointing upwards, one downwards and one on each side. To keep the satellite on track, the two opposing cannons are shot at the same time. Which pair of cannons will be used the least?



A) 1

B) 2

C) 3

D) All cannons will be used equally often.

Question 11 (MC)

Two asteroids with different masses $m_1 < m_2$ orbit around each other according to Newton's laws of motion. Let v_1 be the speed with which the lighter asteroid orbits around the heavier asteroid. Vice versa, let v_2 be the speed with which the heavier asteroid orbits around the lighter asteroid. Assuming that the orbits are circular, what is the relation of the two speeds?

A) $v_1 < v_2$ B) $v_1 > v_2$

C) $v_1 = v_2$ D) We don't know.

Question 12 (MC)

What is the maximal height from which you can drink from a glass of water using a straw?

A) 0.5 m B) 1 m C) 2 m D) 5 m E) 10 m

Question 13 (MC)

Newton stands on the Tower of Pisa and the wind is blowing due north with speed v_w. He throws an apple horizontally due west with speed v_a. We model the air resistance with Stokes' law (friction proportional to the speed), which results in a terminal fall speed of $-\frac{mg}{\beta}$. The Tower of Pisa is veeery high, therefore we may assume that the apple has reached a constant speed before impact. Which speed would that be?

A) $-\frac{mg}{\beta}$ downwards

B) $-\frac{mg}{\beta}$ downwards and $-\frac{mg}{\beta}$ due south

C) $-\frac{mg}{\beta}$ downwards and $-\frac{mg}{\beta}$ due north

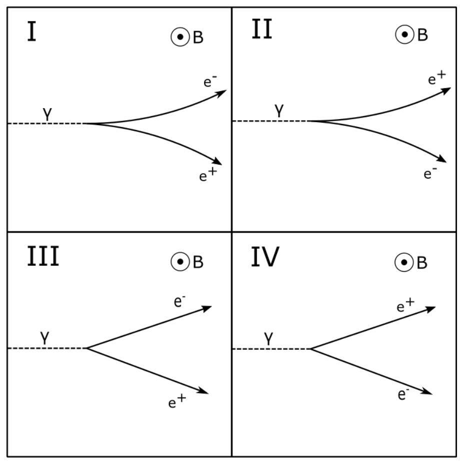
D) $-\frac{mg}{\beta}$ downwards and $-\frac{mg}{\beta}$ due west

E) $-\frac{mg}{\beta}$ downwards and v_a due west

F) $-\frac{mg}{\beta}$ downwards and v_w due north

Question 14 (MC)

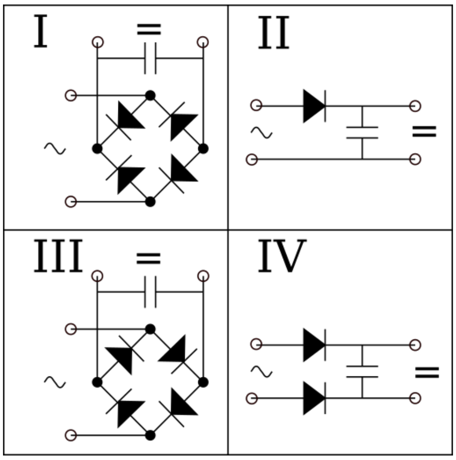
In a particle detector the decay of a photon γ into an electron-positron-pair is measured. The positron is the antiparticle of the electron, with the same mass as the electron but a positive charge. Which of the following traces can be measured in the particle detector? Note that there is a homogeneous magnetic field pointing out of the drawing plane.



A) I B) II C) III D) IV

Question 15 (MTF)

Which of the following circuits can be used to transform an AC-input (∼) to a DC (=) signal? Hint: The diode allows the current to flow only in the direction of the arrow. The capacity is used to smoothen the signal.



A) I B) II C) III D) IV

Question 16 (MC)

Emmy built a parallel-plate capacitor, consisting of two metal plates at a distance d of each other. Now she wants to increase the capacity. Which of the following options results in the largest capacity?

A) Inserting a (dielectric) plastic plate with $\epsilon_r = 2$ and thickness d between the capacitor plates.

B) Inserting a metal plate with thickness $\frac{d}{2}$ in between the two original plates, such that it is lying on one of them.

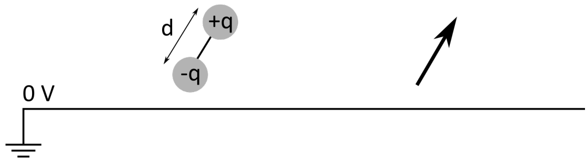
C) Inserting a metal plate with thickness $\frac{d}{2}$ in-between the two original plates (parallel, such that it is not touching either of them).

D) All listed measures decrease the capacity.

E) All listed measures increase the capacity the same.

Question 17 (MC)

A dipole consists of two charges $+q$ and $-q$ at a fixed distance d from each other. It can also be represented by an arrow (at the right in the picture below). The dipole is placed above a grounded metallic plate (zero-potential). What happens?



A) The dipole turns around itself with a constant angular velocity.

B) The dipole does not move.

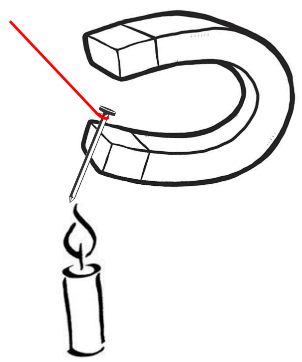
C) The dipole turns around itself, until it is parallel to the metal plate.

D) The dipole turns around itself, until it is perpendicular to the metal plate.

E) The dipole moves parallel to the metal plate.

Question 18 (MC)

A nail is attached to a string and attracted by a magnet. A candle is placed below the nail, such that the magnet is not heated up. Furthermore, the string is not attached to the ceiling directly over the candle (see picture). What happens?



A) Nothing.

B) The nail is attracted even stronger by the magnet.

C) The nail detaches from the magnet.

D) The nail oscillates between a phase where it is attached, and one where it is unattached.

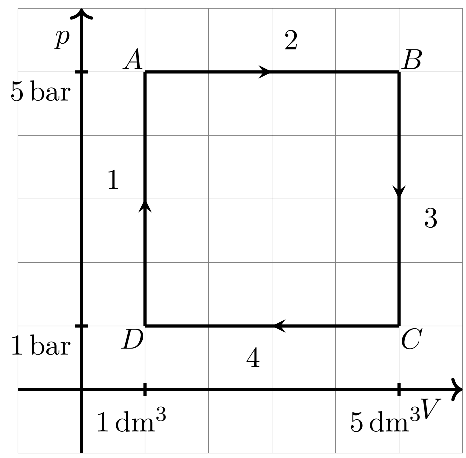
Question 19 (MC)

On a sunny summer day, Lea wants to go riding the bicycle. Outside it is 25 °C. To have an optimal ride, she first checks the pressure of the tires and notices that in the front tire it is only 2 bar instead of the optimal 4 bar. To pump the tire she uses a basic hand pump, which aspires the ambient air. How many times must she pump to reach the optimal pressure? The tire has a volume of $2800\,\text{cm}^3$ and the pump has a volume of $700\,\text{cm}^3$.

A) 2 B) 4 C) 6 D) 8 E) 10

Question 20 (MTF)

A machine, filled with an ideal gas, undergoes the following cyclic process. Which of the following statements are correct?



A) The net work is 800 J.

B) The inner energy of the ideal gas after a cycle is the same as before.

C) The entropy of the gas increased after one cycle.

D) During the process 1 there is no heat exchange.

E) The lowest temperature is reached at point D.

Question 21 (MC)

What is the air pressure p_c at the ground in the center of a tropical cyclone? v_m is the maximal tangential velocity, p_m the air pressure at the ground level vertically below the place where the cyclone has speed v_m. $R_d = 287\,\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$ is the gas constant for dry air and T_b the temperature at the bottom of the cloud boundary of the cyclone.

A) $p_c = p_m \exp\left(-\frac{v_m}{2R_dT_b}\right)$ B) $p_c = p_m \exp\left(-\frac{v_m^2}{2R_dT_b}\right)$

C) $p_c = p_m \exp\left(-\frac{v_m^2}{2R_dT_b^3}\right)$ D) $p_c = p_m \exp\left(-\frac{v_m}{2R_dT_b^3}\right)$

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Physics Olympiad

Second Round

online, 13 January 2021

Theory	:	**3 problems**
Duration	:	120 minutes
Total	:	48 points (15 + 17 + 16)
Authorized material	:	Calculator without database Writing and drawing material One A4 double-sided handwritten page of notes

Good luck!

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Theoretical Problems

Duration: 120 minutes
Marks: 48 points

Start each problem on a new sheet in order to ease the correction. Label the sheets with your name and the number of the problem. Furthermore, number your sheets.

General hint: The problems consist of partially independent problem parts, so if you get stuck, it is a good idea to read further ahead and to continue with an easier part.

Natural constants

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Problem 1.1: Rotating table *(15 points)*

In this question we will analyze a round table with a massless guide rail. The guide rail is fixed on the table and a mass m can move along the guide rail without any friction. The mass is connected with the center of the table by a spring with spring constant k and initial length l_0.

Part A. Along the diameter *(4 points)*

The guide rail is fixed along the diameter on the table, as depicted in illustration 1.1.1. The distance x denotes the displacement from the equilibrium position of the spring.

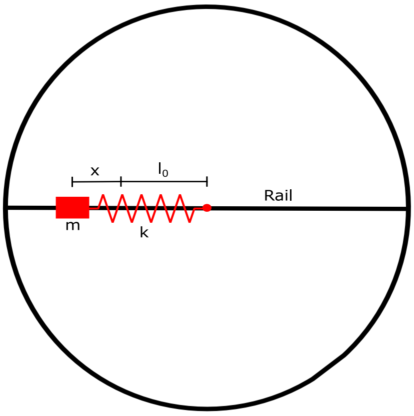

Figure 1.1.1: Table with a spring

i. (1 pt.) What is the angular frequency of the oscillation if the table is not rotating?

We now place the mass m at rest at a distance l_0 of the center of the table, so that the spring is not under any tension. Then the table is turned at constant angular speed ω_0.

ii. (1 pt.) What happens if the angular speed ω_0 is very large?

iii. (1 pt.) What is the maximal value for ω_0 where the mass m oscillates?

iv. (1 pt.) Is the angular frequency of this oscillation larger or smaller than in the case when the table is not rotating?

Part B. Along the chord *(4 points)*

The table is brought to a stop and the guide rail is fixed at a different position. It is now at a chord of the table as depicted in illustration 1.1.2. The distance of the guide rail to the center of the table is l_0. This guarantees that the spring can return to a rest position where there is no tension on the spring. In the following questions we will call x the deflection of the mass m from the middle of the guide rail. d shall be the distance of the mass to the center of the table. And θ the angle between the spring and the direct line between center of the table and center of the guide rail.

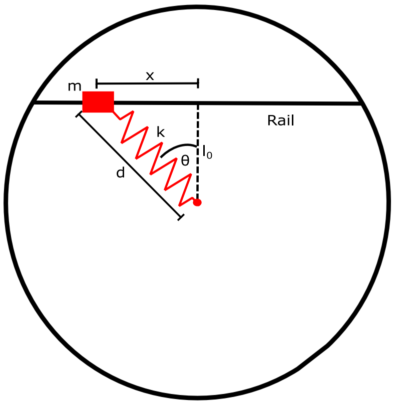

Figure 1.1.2: Table with a guide rail along the chord

i. (2 pt.) How large is the force that pushes the mass back to the initial rest position? Your result should be expressed as a function of deflection x and initial spring length l_0.

ii. (2 pt.) Calculate the amount of energy stored in the spring as a function of x and l_0.

Part C. Equilibrium positions *(7 points)*

We once again start turning the table at an angular speed ω_T.

i. (2 pt.) Draw a scheme with all forces that act upon the mass m when it is not in an equilibrium position. Which condition for the forces must hold for it to be an equilibrium position?

ii. (1 pt.) Find an expression for the total force acting on mass m as a function of the distance d and angle θ.

iii. (2 pt.) How many equilibrium positions are there? Calculate their distance to the center of the table as a function of k, ω_T and l_0.

iv. (2 pt.) State whether these equilibrium positions are stable or unstable equilibria. No justification or explanation is needed.

Problem 1.2: Thermal convection *(17 points)*

Thermal convection describes the phenomenon of a flow occurring in a gas due to temperature differences. The best-known example of thermal convection is the rise of warm air in the Earth's atmosphere. In this problem, we will investigate this phenomenon in more detail using the ideal gas model.

For the following problems, we always assume that the atmosphere is a diatomic ideal gas with a molar mass M, consisting of a single atomar species.

Part A. Ideal gas *(3.5 points)*

i. (0.5 pt.) Name one assumption that applies to the ideal gas at the particle level.

ii. (1.5 pt.) Find a general expression for the density ρ of the ideal gas as a function of p, T and M.

iii. (0.5 pt.) How many degrees of freedom does a diatomic ideal gas have?
Hint: The vibrational degrees of freedom are frozen and do not contribute any energy.

iv. (1 pt.) Infer the numerical value of the adiabatic coefficient γ for a diatomic gas.
Note: If you do not find the value, use the value $4/3$ *for the numerical subtasks.*

Part B. Adiabatic ascent *(3 points)*

To derive a condition for when convection can occur, consider a small package of air in the atmosphere at temperature T_0 and pressure p_0. The air package should now rise adiabatically, i.e. without heat exchange with the environment, by a distance Δr.

i. (1.5 pt.) Find the temperature T after the ascent by Δr as a function of T_0, p_0, γ and of the pressure p after the ascent.

ii. (1.5 pt.) Assume that the distance Δr is very small, so that the pressure Δp changes very little. What is the temperature change ΔT as a function of Δp?
Hint: Use the approximation $(1+x)^\alpha \approx 1 + \alpha x$ *for* $x \ll 1$.

Part C. Convection condition *(4.5 points)*

We now assume that our air package is in equilibrium with the ambient air at the beginning, i.e. the ambient air and the air package have the same temperature T_0, the same pressure p_0 and the same density $\rho_0 = \rho_0'$. The ambient air also has a fixed temperature gradient $\frac{\Delta T'}{\Delta r'}$ and pressure gradient $\frac{\Delta p'}{\Delta r'}$. Due to a disturbance, the air package rises adiabatically by a very small distance Δr, while always remaining in pressure equilibrium with the ambient air.

i. (1 pt.) What condition must hold for the densities ρ' and ρ after the ascent by Δr so that the air package can continue to ascend and a convection current is created?

ii. (1.5 pt.) Infer a condition for ΔT and $\Delta T'$. Justify your answer.

iii. (1 pt.) From this, deduce the convection condition

$$\frac{\Delta T'}{\Delta r'} < \left(1 - \frac{1}{\gamma}\right) \frac{T_0}{p_0} \frac{\Delta p'}{\Delta r'} \tag{1.2.1}$$

where γ is the adiabatic coefficient.

iv. (1 pt.) Suppose we have normal conditions on the Earth's surface. State whether a convection flow is to be expected for the following gradients $\frac{\Delta p'}{\Delta r'} = -0.1\,\text{bar}\cdot\text{km}^{-1}$, $\frac{\Delta T'}{\Delta r'} = -5\,°\text{C}\cdot\text{km}^{-1}$.

Part D. Temperature distribution *(6 points)*

We now want to find an atmosphere temperature curve so that convection is continuously possible.

i. (1 pt.) Suppose we have a column of air of height h, where the density ρ of the air in the column is constant. What is the hydrostatic pressure at the bottom of the air column?

ii. (1 pt.) In the atmosphere, the density $\rho(h)$ of the air decreases with increasing atmospheric height h, so we consider only a very small column of air of length Δr. What is the pressure gradient $\frac{\Delta p}{\Delta r}$ at height h?

For the following subtasks we assume that equality holds in the convection condition (1.2.2).

$$\frac{\Delta T}{\Delta r} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{p} \frac{\Delta p}{\Delta r} \tag{1.2.2}$$

iii. (1 pt.) Assume that we have a temperature gradient large enough that the inequality in equation (1.2.1) holds. Describe the process in the atmosphere that causes equation (1.2.2) to hold after a certain time approximately.

iv. (2 pt.) With this additional assumption, we can determine the temperature distribution in the atmosphere. Find an expression for the temperature at height h as a function of g, γ, R and of the temperature T_0 on the Earth's surface.
Hint: Use equation (1.2.2) *and the results from subtasks D ii. and A ii.*

v. (1 pt.) Assume the temperature outside the Earth's atmosphere is $0\,\mathrm{K}$. The temperature at the Earth's surface is $298.15\,\mathrm{K}$. Use this to calculate the height of the Earth's atmosphere. Use $28\,\mathrm{g}\cdot\mathrm{mol}^{-1}$ for the molar mass M of air.

Problem 1.3: Firework *(16 points)*

Standing across a building with a façade of corrugated iron, the following phenomenon can be observed during a firework display (see figure 1.3.1): Shortly after the bang of a detonating firework K, a person P hears a short buzzing sound from an opposite wall W made of a façade of corrugated iron.
The goal of this problem is to study this phenomenon and to derive expressions for the observed frequency of the buzzing sound.

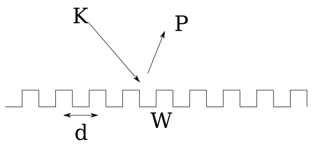

Figure 1.3.1: Situation with the corrugated iron

Part A. The bang *(1.5 points)*

i. (1.5 pt.) How does the propagation of the shock wave after hitting the corrugating iron differ from the reflection on a flat wall?

To simplify the calculations, we examine the phenomenon with a simpler model: We substitute the corrugated iron with very thin periodically arranged bars, see figure 1.3.2.

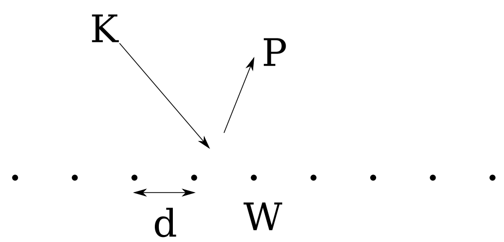

Figure 1.3.2: Simplified model with bars.

In the following problem parts we assume that both the distance from the firework K to the wall W and the distance from the wall W to the person P are large (compared to the dimensions of the wall W).
In the following, we will examinate the problem using two different approaches.

Part B. The pulse cascade *(6.5 points)*

In this part, we will model the bang caused by the firework as a shock wave of very short duration.

i. (2 pt.) First, we consider a special configuration in which the firework explodes at the extension of the wall, see figure 1.3.3. The shock wave of the bang is scattered at each bar. How long is the respective time difference Δt_1, Δt_2 and Δt_3 between the scattered waves of two neighbouring bars for the three different persons P_1, P_2 and P_3?

ii. (1 pt.) How is the buzzing sound created?

iii. (1.5 pt.) What frequencies f_1, f_2 and f_3 does the buzzing sound have for persons P_1, P_2 and P_3? Justify your answer without using equation (1.3.4).

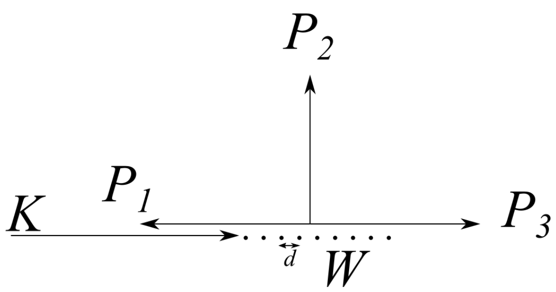

Figure 1.3.3: Special configuration with 3 persons.

iv. (2 pt.) For general angles of incidence and reflection α and β (see figure 1.3.5) the following frequency can be heard at P:

$$f(\alpha,\beta) = \frac{f_d}{\cos(\alpha)+\cos(\beta)}. \tag{1.3.4}$$

Derive this equation and calculate f_d.

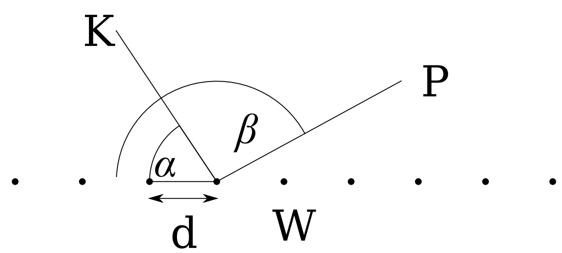

Figure 1.3.5: General configuration.

Part C. The waves *(4 points)*

A bang can also be modelled as a superposition of many planar waves with different frequencies. In this problem part we will now look at a planar wave with a concrete frequency f in the spectrum of the bang, which is scattered at the bars, i.e. we are no longer looking explicitly at a bang.

i. (1 pt.) What condition must be fulfilled for a person P to hear the frequency f? What can be said about the path difference?

ii. (2 pt.) With what angle of reflection β (see figure 1.3.5) can a person hear this frequency f? Calculate it for a general angle of incidence α.

iii. (1 pt.) Compare your result with equation (1.3.4).

Part D. The speed of sound *(4 points)*

i. (4 pt.) In this section we want to determine the speed of sound starting at the phenomenon from part A. Let $\alpha = 45°$ be the incident angle (see figure 1.3.5) and $d = 20\,\text{cm}$ be the distance between bars. The frequency for different angles of reflection β is measured and then listed in table 1.3.6. Plot the measurements in a suitable graph and determine the speed of sound c using these measurements.
Hint: You may use equation (1.3.4), *and should you not have calculated f_d, use $f_d = \pi \frac{c}{d}$.*

β/°	f/Hz
0	930
30	1150
60	1390
90	2410
120	7450

Table 1.3.6: Measurements of the frequency f for different angles of reflection β.

Multiple Choice: Lösungen

	A)	B)	C)	D)	E)	F)
Frage 1	□	□	□	■		
Frage 2	■	□	■	■		
Frage 3	□	□	□	□	■	□
Frage 4	■	□	□	□	□	
Frage 5	□	□	■	□		
Frage 6	□	□	□	■		
Frage 7	□	□	■	□		
Frage 8	□	□	■	□		
Frage 9	□	□	□	■	□	
Frage 10	□	□	■	□		
Frage 11	□	■	□	□		
Frage 12	□	□	□	□	■	
Frage 13	□	□	□	□	□	■
Frage 14	■	□	□	□		
Frage 15	■	■	□	□		
Frage 16	□	□	□	□	■	
Frage 17	□	□	□	■	□	
Frage 18	□	□	□	■		
Frage 19	□	□	□	■	□	
Frage 20	□	■	□	□	■	
Frage 21	□	■	□	□		

Physics Olympiad

Second Round

online, 13 January 2021

Theory	**:**	**3 problems**
Duration	:	120 minutes
Total	:	48 points (15 + 17 + 16)
Authorized material	:	Calculator without database Writing and drawing material One A4 double-sided handwritten page of notes

Good luck!

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Theoretical Problems

Duration: 120 minutes
Marks: 48 points

Start each problem on a new sheet in order to ease the correction. Label the sheets with your name and the number of the problem. Furthermore, number your sheets.

General hint: The problems consist of partially independent problem parts, so if you get stuck, it is a good idea to read further ahead and to continue with an easier part.

Natural constants

Hyperfine transition frequency of caesium	$\Delta\nu_{\mathrm{Cs}}$	$=$	$9\,192\,631\,770$	$\mathrm{s^{-1}}$
Speed of light in vacuum	c	$=$	$299\,792\,458$	$\mathrm{m\cdot s^{-1}}$
Planck constant	h	$=$	$6.626\,070\,15\times10^{-34}$	$\mathrm{kg\cdot m^2\cdot s^{-1}}$
Elementary charge	e	$=$	$1.602\,176\,634\times10^{-19}$	$\mathrm{A\cdot s}$
Boltzmann constant	k_{B}	$=$	$1.380\,649\times10^{-23}$	$\mathrm{kg\cdot m^2\cdot K^{-2}\cdot s^{-2}}$
Avogadro constant	N_{A}	$=$	$6.022\,140\,76\times10^{23}$	$\mathrm{mol^{-1}}$
Luminous efficacy	K_{cd}	$=$	683	$\mathrm{cd\cdot kg\cdot m^2\cdot s^3}$
Magnetic constant	μ_0	$=$	$4\pi\times10^{-7}$	$\mathrm{kg\cdot m\cdot A^{-2}\cdot s^{-2}}$
Electric constant	ε_0	$\approx$	$8.854\,187\,82\times10^{-12}$	$\mathrm{A^2\cdot s^4\cdot kg^{-1}\cdot m^{-3}}$
Gas constant	R	$\approx$	$8.314\,462\,618$	$\mathrm{kg\cdot m^2\cdot K^{-1}\cdot mol^{-1}\cdot s^{-2}}$
Stefan-Boltzmann constant	σ	$\approx$	$5.670\,374\,419\times10^{-8}$	$\mathrm{kg\cdot K^{-4}\cdot s^{-3}}$
Gravitational constant	G	$=$	$6.674\,30(15)\times10^{-11}$	$\mathrm{m^3\cdot kg^{-1}\cdot s^{-2}}$
Electron mass	m_{e}	$=$	$9.109\,383\,701\,5(28)\times10^{-31}$	kg
Neutron mass	m_{n}	$=$	$1.674\,927\,498\,04(95)\times10^{-27}$	kg
Proton mass	m_{p}	$=$	$1.672\,621\,923\,69(51)\times10^{-27}$	kg
Standard acceleration of gravity	g_{n}	$=$	$9.806\,65$	$\mathrm{m\cdot s^{-2}}$

Problem 1.1: Rotating table *(15 points)*

In this question we will analyze a round table with a massless guide rail. The guide rail is fixed on the table and a mass m can move along the guide rail without any friction. The mass is connected with the center of the table by a spring with spring constant k and initial length l_0.

Part A. Along the diameter *(4 points)*

The guide rail is fixed along the diameter on the table, as depicted in illustration 1.1.1. The distance x denotes the displacement from the equilibrium position of the spring.

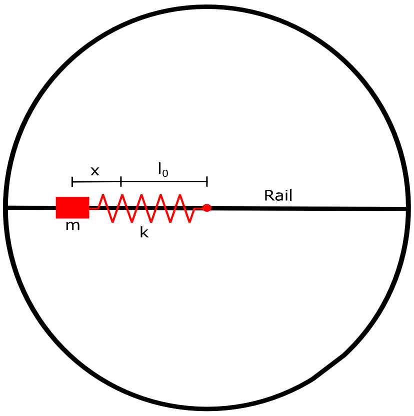

Figure 1.1.1: Table with a spring

i. (1 pt.) What is the angular frequency of the oscillation if the table is not rotating?

We now place the mass m at rest at a distance l_0 of the center of the table, so that the spring is not under any tension. Then the table is turned at constant angular speed ω_0.

ii. (1 pt.) What happens if the angular speed ω_0 is very large?

iii. (1 pt.) What is the maximal value for ω_0 where the mass m oscillates?

iv. (1 pt.) Is the angular frequency of this oscillation larger or smaller than in the case when the table is not rotating?

Part B. Along the chord *(4 points)*

The table is brought to a stop and the guide rail is fixed at a different position. It is now at a chord of the table as depicted in illustration 1.1.2. The distance of the guide rail to the center of the table is l_0. This guarantees that the spring can return to a rest position where there is no tension on the spring. In the following questions we will call x the deflection of the mass m from the middle of the guide rail. d shall be the distance of the mass to the center of the table. And θ the angle between the spring and the direct line between center of the table and center of the guide rail.

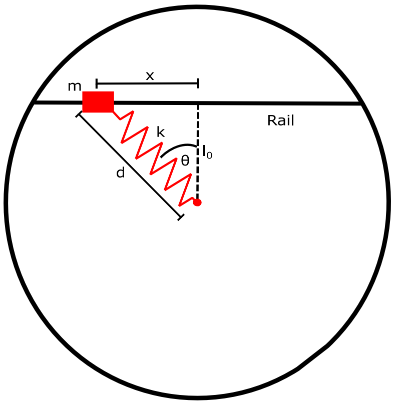

Figure 1.1.2: Table with a guide rail along the chord

i. (2 pt.) How large is the force that pushes the mass back to the initial rest position? Your result should be expressed as a function of deflection x and initial spring length l_0.

ii. (2 pt.) Calculate the amount of energy stored in the spring as a function of x and l_0.

Part C. Equilibrium positions *(7 points)*

We once again start turning the table at an angular speed ω_T.

i. (2 pt.) Draw a scheme with all forces that act upon the mass m when it is not in an equilibrium position. Which condition for the forces must hold for it to be an equilibrium position?

ii. (1 pt.) Find an expression for the total force acting on mass m as a function of the distance d and angle θ.

iii. (2 pt.) How many equilibrium positions are there? Calculate their distance to the center of the table as a function of k, ω_T and l_0.

iv. (2 pt.) State whether these equilibrium positions are stable or unstable equilibria. No justification or explanation is needed.

Problem 1.2: Thermal convection *(17 points)*

Thermal convection describes the phenomenon of a flow occurring in a gas due to temperature differences. The best-known example of thermal convection is the rise of warm air in the Earth's atmosphere. In this problem, we will investigate this phenomenon in more detail using the ideal gas model.

For the following problems, we always assume that the atmosphere is a diatomic ideal gas with a molar mass M, consisting of a single atomar species.

Part A. Ideal gas *(3.5 points)*

i. (0.5 pt.) Name one assumption that applies to the ideal gas at the particle level.

ii. (1.5 pt.) Find a general expression for the density ρ of the ideal gas as a function of p, T and M.

iii. (0.5 pt.) How many degrees of freedom does a diatomic ideal gas have?
Hint: The vibrational degrees of freedom are frozen and do not contribute any energy.

iv. (1 pt.) Infer the numerical value of the adiabatic coefficient γ for a diatomic gas.
Note: If you do not find the value, use the value $4/3$ *for the numerical subtasks.*

Part B. Adiabatic ascent *(3 points)*

To derive a condition for when convection can occur, consider a small package of air in the atmosphere at temperature T_0 and pressure p_0. The air package should now rise adiabatically, i.e. without heat exchange with the environment, by a distance Δr.

i. (1.5 pt.) Find the temperature T after the ascent by Δr as a function of T_0, p_0, γ and of the pressure p after the ascent.

ii. (1.5 pt.) Assume that the distance Δr is very small, so that the pressure Δp changes very little. What is the temperature change ΔT as a function of Δp?
Hint: Use the approximation $(1+x)^\alpha \approx 1+\alpha x$ *for* $x \ll 1$.

Part C. Convection condition *(4.5 points)*

We now assume that our air package is in equilibrium with the ambient air at the beginning, i.e. the ambient air and the air package have the same temperature T_0, the same pressure p_0 and the same density $\rho_0 = \rho_0'$. The ambient air also has a fixed temperature gradient $\frac{\Delta T'}{\Delta r'}$ and pressure gradient $\frac{\Delta p'}{\Delta r'}$. Due to a disturbance, the air package rises adiabatically by a very small distance Δr, while always remaining in pressure equilibrium with the ambient air.

i. (1 pt.) What condition must hold for the densities ρ' and ρ after the ascent by Δr so that the air package can continue to ascend and a convection current is created?

ii. (1.5 pt.) Infer a condition for ΔT and $\Delta T'$. Justify your answer.

iii. (1 pt.) From this, deduce the convection condition

$$\frac{\Delta T'}{\Delta r'} < \left(1 - \frac{1}{\gamma}\right) \frac{T_0}{p_0} \frac{\Delta p'}{\Delta r'} \tag{1.2.1}$$

where γ is the adiabatic coefficient.

iv. (1 pt.) Suppose we have normal conditions on the Earth's surface. State whether a convection flow is to be expected for the following gradients $\frac{\Delta p'}{\Delta r'} = -0.1\,\mathrm{bar \cdot km^{-1}}$, $\frac{\Delta T'}{\Delta r'} = -5\,^\circ\mathrm{C \cdot km^{-1}}$.

Part D. Temperature distribution *(6 points)*

We now want to find an atmosphere temperature curve so that convection is continuously possible.

i. (1 pt.) Suppose we have a column of air of height h, where the density ρ of the air in the column is constant. What is the hydrostatic pressure at the bottom of the air column?

ii. (1 pt.) In the atmosphere, the density $\rho(h)$ of the air decreases with increasing atmospheric height h, so we consider only a very small column of air of length Δr. What is the pressure gradient $\frac{\Delta p}{\Delta r}$ at height h?

For the following subtasks we assume that equality holds in the convection condition (1.2.2).

$$\frac{\Delta T}{\Delta r} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{p} \frac{\Delta p}{\Delta r} \tag{1.2.2}$$

iii. (1 pt.) Assume that we have a temperature gradient large enough that the inequality in equation (1.2.1) holds. Describe the process in the atmosphere that causes equation (1.2.2) to hold after a certain time approximately.

iv. (2 pt.) With this additional assumption, we can determine the temperature distribution in the atmosphere. Find an expression for the temperature at height h as a function of g, γ, R and of the temperature T_0 on the Earth's surface.
Hint: Use equation (1.2.2) *and the results from subtasks D ii. and A ii.*

v. (1 pt.) Assume the temperature outside the Earth's atmosphere is $0\,\mathrm{K}$. The temperature at the Earth's surface is $298.15\,\mathrm{K}$. Use this to calculate the height of the Earth's atmosphere. Use $28\,\mathrm{g}\cdot\mathrm{mol}^{-1}$ for the molar mass M of air.

Problem 1.3: Firework (16 points)

Standing across a building with a façade of corrugated iron, the following phenomenon can be observed during a firework display (see figure 1.3.1): Shortly after the bang of a detonating firework K, a person P hears a short buzzing sound from an opposite wall W made of a façade of corrugated iron.
The goal of this problem is to study this phenomenon and to derive expressions for the observed frequency of the buzzing sound.

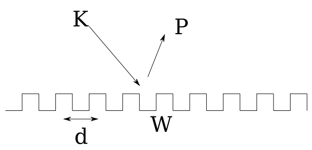

Figure 1.3.1: Situation with the corrugated iron

Part A. The bang (1.5 points)

i. (1.5 pt.) How does the propagation of the shock wave after hitting the corrugating iron differ from the reflection on a flat wall?

To simplify the calculations, we examine the phenomenon with a simpler model: We substitute the corrugated iron with very thin periodically arranged bars, see figure 1.3.2.

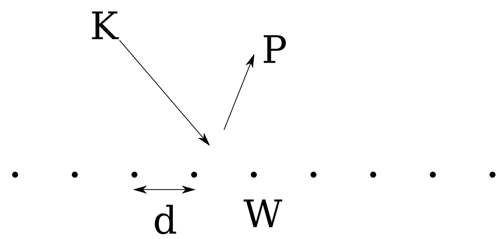

Figure 1.3.2: Simplified model with bars.

In the following problem parts we assume that both the distance from the firework K to the wall W and the distance from the wall W to the person P are large (compared to the dimensions of the wall W).
In the following, we will examinate the problem using two different approaches.

Part B. The pulse cascade (6.5 points)

In this part, we will model the bang caused by the firework as a shock wave of very short duration.

i. (2 pt.) First, we consider a special configuration in which the firework explodes at the extension of the wall, see figure 1.3.3. The shock wave of the bang is scattered at each bar. How long is the respective time difference Δt_1, Δt_2 and Δt_3 between the scattered waves of two neighbouring bars for the three different persons P_1, P_2 and P_3?

ii. (1 pt.) How is the buzzing sound created?

iii. (1.5 pt.) What frequencies f_1, f_2 and f_3 does the buzzing sound have for persons P_1, P_2 and P_3? Justify your answer without using equation (1.3.4).

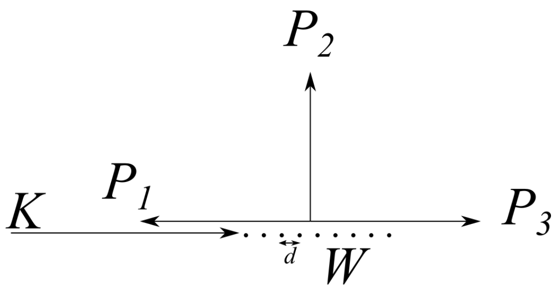

Figure 1.3.3: Special configuration with 3 persons.

iv. (2 pt.) For general angles of incidence and reflection α and β (see figure 1.3.5) the following frequency can be heard at P:

$$f(\alpha, \beta) = \frac{f_d}{\cos(\alpha) + \cos(\beta)}. \tag{1.3.4}$$

Derive this equation and calculate f_d.

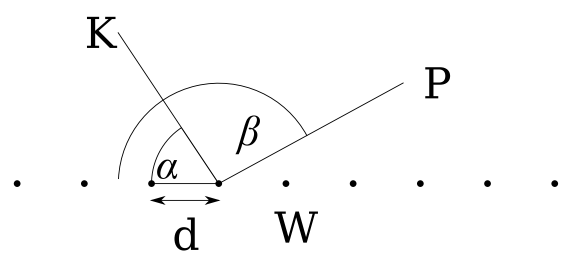

Figure 1.3.5: General configuration.

Part C. The waves (4 points)

A bang can also be modelled as a superposition of many planar waves with different frequencies. In this problem part we will now look at a planar wave with a concrete frequency f in the spectrum of the bang, which is scattered at the bars, i.e. we are no longer looking explicitly at a bang.

i. (1 pt.) What condition must be fulfilled for a person P to hear the frequency f? What can be said about the path difference?

ii. (2 pt.) With what angle of reflection β (see figure 1.3.5) can a person hear this frequency f? Calculate it for a general angle of incidence α.

iii. (1 pt.) Compare your result with equation (1.3.4).

Part D. The speed of sound *(4 points)*

i. (4 pt.) In this section we want to determine the speed of sound starting at the phenomenon from part A. Let $\alpha = 45°$ be the incident angle (see figure 1.3.5) and $d = 20\,\text{cm}$ be the distance between bars. The frequency for different angles of reflection β is measured and then listed in table 1.3.6. Plot the measurements in a suitable graph and determine the speed of sound c using these measurements.
Hint: You may use equation (1.3.4)*, and should you not have calculated f_d, use $f_d = \pi \frac{c}{d}$.*

$\beta/°$	f/Hz
0	930
30	1150
60	1390
90	2410
120	7450

Table 1.3.6: Measurements of the frequency f for different angles of reflection β.

Theoretical Problems: solutions

Problem 1.1: Rotating table — 15 pt.

In this question we will analyze a round table with a massless guide rail. The guide rail is fixed on the table and a mass m can move along the guide rail without any friction. The mass is connected with the center of the table by a spring with spring constant k and initial length l_0.

Part A. Along the diameter — 4 pt.

The guide rail is fixed along the diameter on the table, as depicted in illustration 1.1.1. The distance x denotes the displacement from the equilibrium position of the spring.

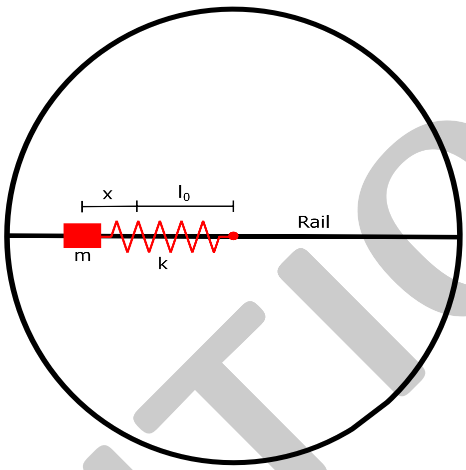

Figure 1.1.1: Table with a spring

i. What is the angular frequency of the oscillation if the table is not rotating? **1 pt.**

$\omega_{\text{spring}} = \sqrt{\frac{k}{m}}$ 1 pt.

Give 0.5 points if the frequency

$$f_{spring} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

instead of the angular frequency is given as a solution.

We now place the mass m at rest at a distance l_0 of the center of the table, so that the spring is not under any tension. Then the table is turned at constant angular speed ω_0.

ii. What happens if the angular speed ω_0 is very large? **1 pt.**

If the angular speed is very high the centrifugal force will always be larger than the spring force, because it is increasing linearly with the distance x as well. Therefore, the mass will move outwards with increasing speed and eventually fall over the edge of the table. 1 pt.

One could also argue that the spring force is smaller than the centripetal force for very high ω_0. In this case also all the points are given.

iii. What is the maximal value for ω_0 where the mass m oscillates? **1 pt.**

At the beginning when $x = 0$, the centrifugal force will always be larger than the spring force, which is zero. The magnitude of both forces increases linearly in distance x with proportionality constants k and $m\omega_0^2$. We have an oscillation in case the spring force will become larger than the centrifugal force at some distance x. Which means we need to have $k > m\omega_0^2$. This leads to

$$\omega_0 < \sqrt{\frac{k}{m}} = \omega_{\text{spring}}$$

1 pt.

This condition can also be found by arguing with the centripetal force. All points are given if the conclusion is correct.

iv. Is the angular frequency of this oscillation larger or smaller than in the case when the table is not rotating? **1 pt.**

The centrifugal force is acting against the spring force, which effectively can be described as if the spring had a smaller spring constant $k' < k$. This means that the new angular frequency ω is smaller than ω_{spring}. 1 pt.

Similarly one can argue that part of the spring force is needed for the centripetal force. Therefore we have an effective spring constant $k' < k$ and we come to the same conclusion as above.

Part B. Along the chord *4 pt.*

The table is brought to a stop and the guide rail is fixed at a different position. It is now at a chord of the table as depicted in illustration 1.1.2. The distance of the guide rail to the center of the table is l_0. This guarantees that the spring can return to a rest position where there is no tension on the spring. In the following questions we will call x the deflection of the mass m from the middle of the guide rail. d shall be the distance of the mass to the center of the table. And θ the angle between the spring and the direct line between center of the table and center of the guide rail.

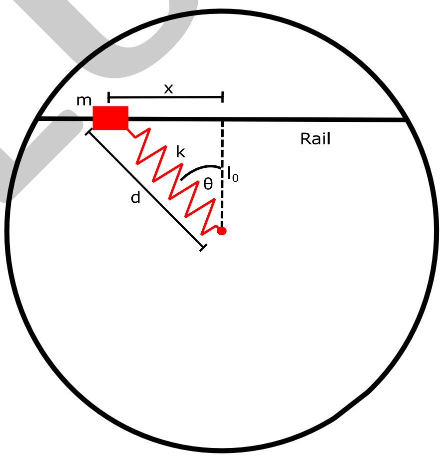

Figure 1.1.2: Table with a guide rail along the chord

i. How large is the force that pushes the mass back to the initial rest position? Your result should be expressed as a function of deflection x and initial spring length l_0. **2 pt.**

The total force from the spring on the mass is

$$F_{\text{tot}} = -k\left(\sqrt{x^2 + l_0^2} - l_0\right)$$

0.5 pt.

Since the mass is fixed on the rail only the component parallel to the rail contributes

$$F = -k\left(\sqrt{x^2 + l_0^2} - l_0\right)\sin(\theta)$$

0.5 pt.

Expressed in term of x, l_0 the sine is

$$\sin(\theta) = \frac{x}{\sqrt{l_0^2 + x^2}}$$

0.5 pt.

Combining the the two equations gives

$$F = -kx\left(1 - \frac{l_0}{\sqrt{l_0^2 + x^2}}\right)$$

0.5 pt.

ii. Calculate the amount of energy stored in the spring as a function of x and l_0. **2 pt.**

The energy in the spring is

$$E = \frac{1}{2}k\left(d - l_0\right)^2$$

0.5 pt.

Expanding the bracket gives

$$E = \frac{1}{2}k\left(d^2 - 2l_0 d + l_0^2\right)$$

0.5 pt.

We can use Pythagoras to substitute the distance d

$$d = \sqrt{x^2 + l_0^2}$$

0.5 pt.

In the end we get

$$E = \frac{1}{2}kx^2 - \left(\frac{\sqrt{l_0^2 + x^2}}{l_0} - 1\right)kl_0^2$$

0.5 pt.

Alternative solution:
The energy in the spring is

$$E = -\int_0^x F(s)\,\mathrm{d}s$$

By evaluating the integral we get the same solution *(0.5 pt*

$$E = \int_0^x ks\left(1 - \frac{l_0}{\sqrt{l_0^2 + s^2}}\right)\mathrm{d}s = \frac{1}{2}kx^2 - \left(\frac{\sqrt{l_0^2 + x^2}}{l_0} - 1\right)kl_0^2$$

~~*(1.5 pt*~~

Part C. Equilibrium positions *7 pt.*

We once again start turning the table at an angular speed ω_T.

i. Draw a scheme with all forces that act upon the mass m when it is not in an equilibrium position. Which condition for the forces must hold for it to be an equilibrium position? **2 pt.**

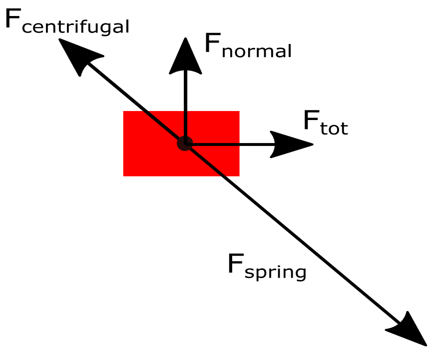

Figure 1.1.3: Force diagram

The force of the spring. 0.5 pt.

The normal force from the rail on the mass. If the spring force is smaller than the centrifugal force the normal force points in a different direction as shown in sketch 1.1.3. 0.5 pt.

The centrifugal force from the rotation. 0.5 pt.

In equilibrium the sum of all the forces is zero. 0.5 pt.

In case this question is solved with the centripetal force, the 0.5 points from the centrifugal force and the 0.5 points from the equilibrium condition are given if one mentions that in equilibrium the total force is equal to the centripetal force.

ii. Find an expression for the total force acting on mass m as a function of the distance d and angle θ. **1 pt.**

The total force on the mass is

$$F_{tot} = m\omega_T^2 d\sin(\theta) - k\left(d - l_0\right)\sin(\theta)$$

For the centrifugal force. 0.5 pt.

For the spring force. 0.5 pt.

In case the problem is solved with the centripetal force, full points are given for

$$F_{tot} = -k\left(d - l_0\right)\sin(\theta)$$

iii. How many equilibrium positions are there? Calculate their distance to the center of the table as a function of k, ω_T and l_0. **2 pt.**

In equilibrium the total force is zero, which means that either $\sin(\theta) = 0$ or the centrifugal force is equal to the spring force

$$m\omega_T^2 d = k\left(d - l_0\right)$$

where d is the distance between the mass and the center of the circle. 0.5 pt.

The same condition is found in case if one says the centripetal force is equal to spring force.

Solving for d gives

$$d = l_0 \frac{k}{k - m\omega_T^2}$$

0.5 pt.

This solution only makes sense in the case $m\omega_T^2 < k$, which means in this case there are three equilibrium points, two at distances d and one at distance l_0. 0.5 pt.

In the case $m\omega_T^2 > k$, there is only the equilibrium point in the middle at distance l_0. 0.5 pt.

iv. State whether these equilibrium positions are stable or unstable equilibria. No justification or explanation is needed. **2 pt.**

First we look at the equilibrium point in the middle ($d = l_0$). At $x = 0$ the total force of the spring is 0 but the centrifugal force is $m\omega_T^2 l_0 > 0$. Therefore for small deviations δx the mass will be pushed away from the equilibrium point. This means the point is unstable. 1 pt.

For the other two points we first note that the spring force and the centrifugal force are both linearly increasing in d but just in opposite directions. The proportionality constants are $m\omega_T^2$ and k. In these two equilibrium points both forces are equal in magnitude and we know $m\omega_T^2 < k$. This means for distances d further from the center the spring force becomes dominant which pushes the mass back to equilibrium. The opposite happens for distances d closer to center than the equilibrium, where the centrifugal is dominant, which pushes the mass out to the equilibrium. Thereby these two equilibrium points are stable. 1 pt.

Problem 1.2: Thermal convection *17 pt.*

Thermal convection describes the phenomenon of a flow occurring in a gas due to temperature differences. The best-known example of thermal convection is the rise of warm air in the Earth's atmosphere. In this problem, we will investigate this phenomenon in more detail using the ideal gas model.

For the following problems, we always assume that the atmosphere is a diatomic ideal gas with a molar mass M, consisting of a single atomar species.

Part A. Ideal gas *3.5 pt.*

i. Name one assumption that applies to the ideal gas at the particle level. **0.5 pt**

Points are given for one of the following statements:

- There isn't any attraction between the molecules.
- The collisions between the molecules are elastic.
- The collisions between the molecules and the wall are elastic.
- The atoms in the gas molecules are point like.

0.5 pt.

ii. Find a general expression for the density ρ of the ideal gas as a function of p, T and M. **1.5 pt**

In the ideal gas law

$$pV = nRT$$

0.5 pt.

We can substitute the number of gas molecules n by the density

$$n = \frac{\rho V}{M}$$

0.5 pt.

After some algebraic transformation we get

$$\rho = \frac{pM}{RT}$$

0.5 pt.

iii. How many degrees of freedom does a diatomic ideal gas have?
Hint: The vibrational degrees of freedom are frozen and do not contribute any energy. **0.5 pt**

We have 3 translational degrees of freedom and 2 rotational degrees of freedom. In total we therefore have 5. 0.5 pt.

iv. Infer the numerical value of the adiabatic coefficient γ for a diatomic gas.
Note: If you do not find the value, use the value $4/3$ for the numerical subtasks. **1 pt.**

We have $\gamma = \frac{c_p}{c_v}$ with $c_p = \frac{f+2}{2}R$ and $c_v = \frac{f}{2}R$, where f are the degree of freedoms. 0.5 pt.

Plugging in the values gives $\gamma = 1.4$. 0.5 pt.

Note: If the adiabatic coefficient is stated directly without reasoning only 0.5 points are given.

Part B. Adiabatic ascent *3 pt.*

To derive a condition for when convection can occur, consider a small package of air in the atmosphere at temperature T_0 and pressure p_0. The air package should now rise adiabatically, i.e. without heat exchange with the environment, by a distance Δr.

i. Find the temperature T after the ascent by Δr as a function of T_0, p_0, γ and of the pressure p after the ascent. **1.5 pt**

From the adiabatic equation in p, T we get

$$p^{\frac{1-\gamma}{\gamma}} T = p_0^{\frac{1-\gamma}{\gamma}} T_0$$

1 pt.

Rearranging the terms gives

$$T = \frac{p_0^{\frac{1-\gamma}{\gamma}} T_0}{p^{\frac{1-\gamma}{\gamma}}}$$

0.5 pt.

Note: The adiabatic equation in p, T can be derived from the standard adiabatic equation in p, V: $pV^\gamma = const.$

$$pV^\gamma = const. \Longrightarrow p^{\frac{1}{\gamma}} V = const.$$

By the ideal gas law we get

$$p^{\frac{1}{\gamma}-1} Tnr = const. \Longrightarrow p^{\frac{1-\gamma}{\gamma}} T = const.$$

ii. Assume that the distance Δr is very small, so that the pressure Δp changes very little. What is the temperature change ΔT as a function of Δp?
Hint: Use the approximation $(1+x)^\alpha \approx 1 + \alpha x$ for $x \ll 1$. **1.5 pt**

From the solution above we have

$$T_0 + \Delta T = p_0^{\frac{1-\gamma}{\gamma}} T_0 \left(p_0 + \Delta p\right)^{-\frac{1-\gamma}{\gamma}} = T_0 \left(1 + \frac{\Delta p}{p_0}\right)^{-\frac{1-\gamma}{\gamma}}$$

0.5 pt.

Using the approximation given in the hint

$$T_0 + \Delta T = T_0 - \frac{1-\gamma}{\gamma} \Delta p \frac{T_0}{p_0}$$

0.5 pt.

We conclude

$$\Delta T = \left(1 - \frac{1}{\gamma}\right) \Delta p \frac{T_0}{p_0}$$

0.5 pt.

Part C. Convection condition *4.5 pt.*

We now assume that our air package is in equilibrium with the ambient air at the beginning, i.e. the ambient air and the air package have the same temperature T_0, the same pressure p_0 and the same density $\rho_0 = \rho_0'$. The ambient air also has a fixed temperature gradient $\frac{\Delta T'}{\Delta r'}$ and pressure gradient $\frac{\Delta p'}{\Delta r'}$. Due to a disturbance, the air package rises adiabatically by a very small distance Δr, while always remaining in pressure equilibrium with the ambient air.

i. What condition must hold for the densities ρ' and ρ after the ascent by Δr so that the air package can continue to ascend and a convection current is created? **1 pt.**

The buoyancy force needs to be higher than the gravitational force, so that the air continues to rise. 0.5 pt.

Mathematically this conditions reads as

$$\rho < \rho'$$

0.5 pt.

Note: Full points are also given if the solution is stated directly.

ii. Infer a condition for ΔT and $\Delta T'$. Justify your answer. **1.5 pt**

We use the formula derived for the density of the ideal gas in A ii.

$$\frac{M\,(p_0 + \Delta p)}{R\,(T_0 + \Delta T)} = \rho < \rho' \frac{M\,(p_0' + \Delta p')}{R\,(T_0' + \Delta T')}$$

0.5 pt.

Since the pressure is in equilibrium $\Delta p' = \Delta p$ and $p_0 = p_0'$, $T_0 = T_0'$ we get

$$\frac{1}{T_0 + \Delta T} < \frac{1}{T_0 + \Delta T'}$$

0.5 pt.

We conclude

$$\Delta T > \Delta T'$$

0.5 pt.

Note: If the condition is stated directly without reasoning only 0.5 points are given.

iii. From this, deduce the convection condition

$$\frac{\Delta T'}{\Delta r'} < \left(1 - \frac{1}{\gamma}\right) \frac{T_0}{p_0} \frac{\Delta p'}{\Delta r'} \tag{1.2.1}$$

where γ is the adiabatic coefficient. **1 pt.**

We can use the expression for ΔT derived before

$$\left(1 - \frac{1}{\gamma}\right) \frac{T_0}{p_0} \Delta p = \Delta T < \Delta T'$$

0.5 pt.

Since the pressure is in equilibrium we get $\Delta p = \Delta p'$. Rearranging the terms and dividing by $\Delta r'$ we get the convection condition

$$\left(1 - \frac{1}{\gamma}\right) \frac{T_0}{p_0} \frac{\Delta p'}{\Delta r'} > \frac{\Delta T'}{\Delta r'}$$

0.5 pt.

iv. Suppose we have normal conditions on the Earth's surface. State whether a convection flow is to be expected for the following gradients $\frac{\Delta p'}{\Delta r'} = -0.1\,\mathrm{bar}\cdot\mathrm{km}^{-1}$, $\frac{\Delta T'}{\Delta r'} = -5\,°\mathrm{C}\cdot\mathrm{km}^{-1}$. **1 pt.**

Normal conditions mean $p_0 = 1\,\mathrm{bar}$ and $T_0 = 273.15\,\mathrm{K}$ (or $293.15\,\mathrm{K}$ depending on the exact definition of the normal conditions). 0.5 pt.

For both values of the adiabatic coefficient (1.4 and 4/3) the convection condition is not fulfilled and no convection occurs. 0.5 pt.

For convection to occur, one needs a temperature gradient of about $-10\,°\mathrm{C}\cdot\mathrm{km}^{-1}$.

Part D. Temperature distribution *6 pt.*

We now want to find an atmosphere temperature curve so that convection is continuously possible.

i. Suppose we have a column of air of height h, where the density ρ of the air in the column is constant. What is the hydrostatic pressure at the bottom of the air column? **1 pt.**

The hydrostatic pressure is

$$p = \rho g h$$

1 pt.

ii. In the atmosphere, the density $\rho(h)$ of the air decreases with increasing atmospheric height h, so we consider only a very small column of air of length Δr. What is the pressure gradient $\frac{\Delta p}{\Delta r}$ at height h? **1 pt.**

For a small air package we can assume that the density is constant. So the pressure difference throughout the air package is $\Delta p = -g\rho\Delta r$. Therefore we get

$$\frac{\Delta p}{\Delta r} = -g\rho(h)$$

1 pt.

For the following subtasks we assume that equality holds in the convection condition (1.2.2).

$$\frac{\Delta T}{\Delta r} = \left(1 - \frac{1}{\gamma}\right) \frac{T}{p} \frac{\Delta p}{\Delta r} \tag{1.2.2}$$

iii. Assume that we have a temperature gradient large enough that the inequality in equation (1.2.1) holds. Describe the process in the atmosphere that causes equation (1.2.2) to hold after a certain time approximately. **1 pt.**

In case the temperature drops that rapidly, so that the convection condition is fulfilled, the warm air at the ground starts to rise. Since the warm air is rising the temperature gradient will decrease until we have an equality in the convection condition. 1 pt.

iv. With this additional assumption, we can determine the temperature distribution in the atmosphere. Find an expression for the temperature at height h as a function of g, γ, R and of the temperature T_0 on the Earth's surface.
Hint: Use equation (1.2.2) ***and the results from subtasks D ii. and A ii.*** **2 pt.**

We use equation (1.2.2) and the expression for the pressure gradient to get a differential equation for the temperature gradient

$$\frac{\Delta T}{\Delta r} = -\left(1 - \frac{1}{\gamma}\right)\frac{T}{p} g\rho$$

0.5 pt.

With the solution from question A ii. this can be simplified to

$$\frac{\Delta T}{\Delta r} = -\left(1 - \frac{1}{\gamma}\right) g\frac{M}{R}$$

0.5 pt.

We see that the temperature gradient is a constant independent of the height h. Therefore we get

$$T(h) = T_0 - \left(1 - \frac{1}{\gamma}\right) g\frac{M}{R} h = T_0 - Ah$$

1 pt.

v. Assume the temperature outside the Earth's atmosphere is $0\,\mathrm{K}$. The temperature at the Earth's surface is $298.15\,\mathrm{K}$. Use this to calculate the height of the Earth's atmosphere. Use $28\,\mathrm{g}\cdot\mathrm{mol}^{-1}$ for the molar mass M of air. **1 pt.**

Let h_a be the height of the atmosphere. Therefore we have $0 = -Ah_a + T_0$. This means

$$h_a = \frac{T_0}{A}$$

0.5 pt.

We get a numerical value of $32\,\mathrm{km}$. 0.5 pt.

The value of the constant is

$$A = 9.44\,\mathrm{K}\cdot\mathrm{km}^{-1}$$

In case $\gamma = 4/3$ the numerical values are

$$A = 8.26\,\mathrm{K}\cdot\mathrm{km}^{-1} \textit{ and } r_a = 36\,\mathrm{km}$$

Problem 1.3: Firework *16 pt.*

Standing across a building with a façade of corrugated iron, the following phenomenon can be observed during a firework display (see figure 1.3.1): Shortly after the bang of a detonating firework K, a person P hears a short buzzing sound from an opposite wall W made of a façade of corrugated iron.
The goal of this problem is to study this phenomenon and to derive expressions for the observed frequency of the buzzing sound.

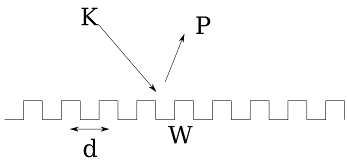

Figure 1.3.1: Situation with the corrugated iron

Part A. The bang *1.5 pt.*

i. How does the propagation of the shock wave after hitting the corrugating iron differ from the reflection on a flat wall? **1.5 pt**

In case of a corrugated iron, the sound wave gets scattered at each front plate. 0.5 pt.

The outgoing waves are cylindrical waves. 0.5 pt.

In case of a plane wall, the reflected sound wave is rather a plane wave. 0.5 pt.

To simplify the calculations, we examine the phenomenon with a simpler model: We substitute the corrugated iron with very thin periodically arranged bars, see figure 1.3.2.

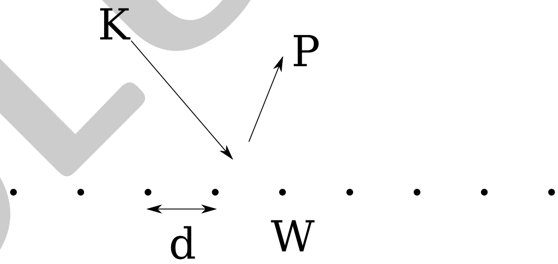

Figure 1.3.2: Simplified model with bars.

In the following problem parts we assume that both the distance from the firework K to the wall W and the distance from the wall W to the person P are large (compared to the dimensions of the wall W).
In the following, we will examinate the problem using two different approaches.

Part B. The pulse cascade *6.5 pt.*

In this part, we will model the bang caused by the firework as a shock wave of very short duration.

i. First, we consider a special configuration in which the firework explodes at the extension of the wall, see figure 1.3.3. The shock wave of the bang is scattered at each bar. How long is the

respective time difference Δt_1, Δt_2 and Δt_3 between the scattered waves of two neighbouring bars for the three different persons P_1, P_2 and P_3? **2 pt.**

In all cases: Two successive bangs origin from the scattering of neighbouring bars. Therefore it suffices to compute the additional propagation distance between neighbouring bars (give these points also in case it is not implicitly mentioned but the following computation correct). 0.5 pt.

For P_1: Consider one bar. To reach the right sided neighbouring bar, the bang needs to travel a distance of d. Then to return to the considered bar, the scattered bang travels another distance d. The additional distance is $2d$, hence the $\Delta t_1 = \frac{2d}{c}$. 0.5 pt.

For P_2: The bang needs again to travel a distance d to reach the next right sided neighbouring bar. Since the scattered bangs travel perpendicular to the corrugated iron to reach P_2 (and P_2 is far away), there is no additional distance. The total path difference is only d, hence the $\Delta t_1 = \frac{d}{c}$. 0.5 pt.

For P_3: In this case, the bang getting scattered at a bar overlaps with the original bang. Therefore all the scattered bangs arrive simultaneously, hence $\Delta t = 0$. 0.5 pt.

ii. How is the buzzing sound created? **1 pt.**

The buzzing sound origins from the cascading of the scattered bangs. 1 pt.

iii. What frequencies f_1, f_2 and f_3 does the buzzing sound have for persons P_1, P_2 and P_3? Justify your answer without using equation (1.3.4).

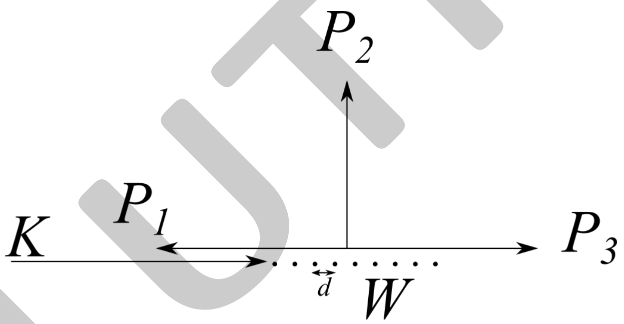

Figure 1.3.3: Special configuration with 3 persons.

1.5 pt

Note: If the equation (1.3.4) is used or of the result is given without explanation, give 0 points! Exception: The general equation (1.3.4) is derived correctly and it is explained how to use it (i.e. what values of α and β inserted).

The frequency is given by $f = \frac{1}{T}$ where T is the period.

For P_1 we get $f = \frac{c}{2d}$. 0.5 pt.

For P_2 we get $f = \frac{c}{d}$. 0.5 pt.

For P_3 we have to be careful: We hear a single bang, because all the scattered bang arrive at the same time. So no particular frequency can be attributed. 0.5 pt.

iv. For general angles of incidence and reflection α and β (see figure 1.3.5) the following frequency can be heard at P:

$$f(\alpha, \beta) = \frac{f_d}{\cos(\alpha) + \cos(\beta)}. \tag{1.3.4}$$

Derive this equation and calculate f_d.

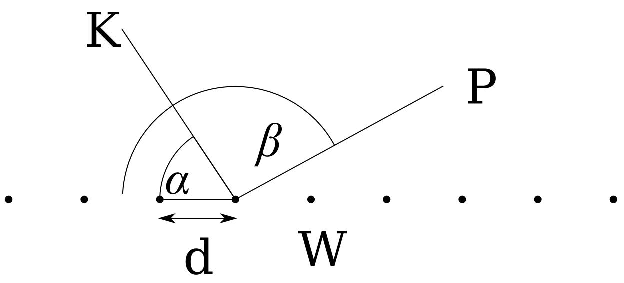

Figure 1.3.5: General configuration.

2 pt.

Note: If the equation (1.3.4) is used or if the result is given without explanation, give 0 points! Exception: The general equation (1.3.4) is derived correctly and it is explained how to use it (i.e. what values of α and β inserted).

The frequency is given by $f = \frac{1}{T}$ where T is the period.

Consider again a bar. The additional path length for the bang to reach the right handed neighbour is $\cos(\alpha)\, d$. 0.5 pt.

The additional path length from the right handed neighbour to P is $\cos(\beta)\, d$. 0.5 pt.

The total temporal delay is therefore $\Delta t = \frac{d(\cos(\alpha)+\cos(\beta))}{c}$ (0.25 points are given for dividing by c and 0.25 points for adding the two path length). 0.5 pt.

We get the frequency by taking the reciprocal value $f = \frac{1}{\Delta t}$.
Therefore we have $f_d = \frac{c}{d}$ (these points are given for the right result even if there is no or a wrong explanation). 0.5 pt.

Part C. The waves *4 pt.*

A bang can also be modelled as a superposition of many planar waves with different frequencies. In this problem part we will now look at a planar wave with a concrete frequency f in the spectrum of the bang, which is scattered at the bars, i.e. we are no longer looking explicitly at a bang.

i. What condition must be fulfilled for a person P to hear the frequency f? What can be said about the path difference? **1 pt.**

The scattered waves from the different bars must interfere constructively. 0.5 pt.

Therefore the difference of the path length of the different scattered waves must be a multiple of the wavelength $\Delta l = m\lambda$, $m \in \mathbb{N}$ (these points are also given if the multiple path length is mentioned in the next task). 0.5 pt.

ii. With what angle of reflection β (see figure 1.3.5) can a person hear this frequency f? Calculate it for a general angle of incidence α. **2 pt.**

For the frequency f we have a wavelength $\lambda = \frac{c}{f}$ (give these points also if it is mentioned in the previous task). 0.5 pt.

The additional path length is $\Delta l = d\,(\cos(\alpha) + \cos(\beta))$. 0.5 pt.

Equating the additional path length with a multiple of λ. 0.5 pt.

Final result: $\beta = \arccos\left(\frac{m\lambda}{d} - \cos(\alpha)\right)$. 0.5 pt.

If only the first order $m = 1$ or $m = -1$ is used, punishment of -0.25 points.

iii. Compare your result with equation (1.3.4). **1 pt.**

For $m = 1$ (give these points only if the computation above is also done with a general $m \in \mathbb{N}$ and don't if $m \pm 1$), 0.5 pt.

and solving for $f = \frac{c}{\lambda}$ we get the same result (give these points even if nothing else in this part C is done but the student realizes that the two approaches are the same). 0.5 pt.

Part D. The speed of sound *4 pt.*

i. In this section we want to determine the speed of sound starting at the phenomenon from part A. Let $\alpha = 45°$ be the incident angle (see figure 1.3.5) and $d = 20\,\text{cm}$ be the distance between bars. The frequency for different angles of reflection β is measured and then listed in table 1.3.6. Plot the measurements in a suitable graph and determine the speed of sound c using these measurements.

Hint: You may use equation (1.3.4), and should you not have calculated f_d, use $f_d = \pi\frac{c}{d}$.

$\beta/°$	f/Hz
0	930
30	1150
60	1390
90	2410
120	7450

Table 1.3.6: Measurements of the frequency f for different angles of reflection β.

Concerning the plot (total 2 points): It does not matter what plot a student draws (i.e. whether it is a β-f plot or a β-c plot. Distribute the points as follows: **4 pt.**

Axis labelled (0.25 points for x and y each). 0.5 pt.

Scale of axis visible and reasonable (0.25 points for x and y each). 0.5 pt.

Both axis drawn with ruler (i.e. straight line). 0.25 pt

Big enough plot. 0.25 pt

Data points correctly drawn (only 0.25 points if one point is clearly wrong and 0 points if two or more are wrong). 0.5 pt.

Punishment of 0.5 points if the data points are connected with a line (no matter whether straight or curved line) but trend line is ok.

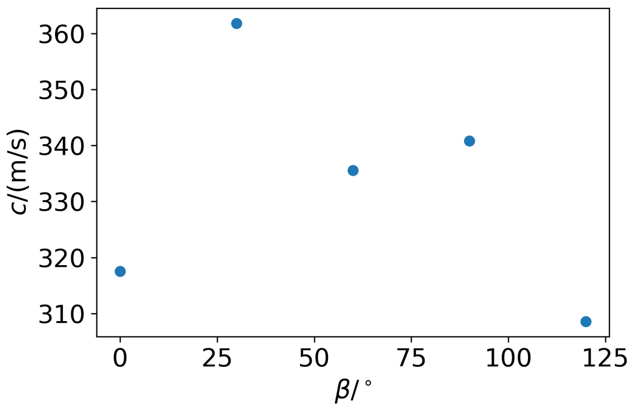

Figure 1.3.7: Computed speed of sound.

Concerning the data evaluation (2 points): There is no punishment if $f_d = \pi \frac{c}{d}$ is used, but the result is simply smaller by a factor of π.

Computation of the speed of sound correctly: $c = df\,(\cos(\alpha) + \cos(\beta))$. 0.5 pt.

Reasonable method to evaluate the data (for example compute for each data point the speed of sound and take the average or guess the average by inserting a trend line in the β-c plot). 1 pt.

Get the correct value of $333\,\mathrm{m \cdot s^{-1}}$ (give points in between the interval $328 - 338\mathrm{m \cdot s^{-1}}$). 0.5 pt.