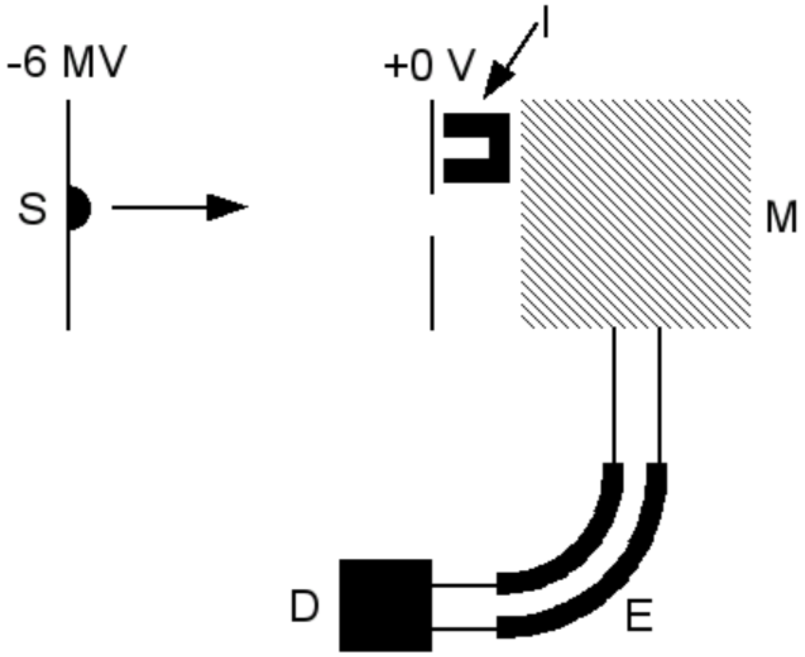
Challenge 4, Electrodynamics: Solution

Accelerator mass spectrometer *12 pt.*

Consider a simple model of a so-called acclerator mass spectrometer, which is used to date moraines of glaciers. To do that, the ratio of the ^{10}Be ($m_{10} = 1.7 \times 10^{-27}\,\mathrm{kg}$) isotope to the stable isotope ^{9}Be is measured (Be = Beryllium). ^{9}Be is contained in the rock material and ^{10}Be is produced in very low concentrations by the cosmic rays hitting the rock surface.

Single negatively charged ^{9}Be and ^{10}Be ions are extracted from a rock sample in the ion source S. The source is on a electrostatic potential of $-6\,\mathrm{MV}$. To the right of the source there is a metal plate with a slit which is held at ground potential ($0\,\mathrm{MV}$). Therefore the ions are accelerated in the direction indicated by the arrow and pass through the slit.



i. Which is the energy and the velocity of the ^{10}Be ions after they have passed the slit? **2 pt.**

For a charged particle passing a gap between two plates on different potential we know $E = qU$ after the passed gap. 0.5 pt.

Let $q = -1e$ because we have single negatively charged particles. $U = -6\,\mathrm{MV}$, thus $E = 9.6 \times 10^{-13}\,\mathrm{J}$. 0.5 pt.

But we also know $E = \frac{m_{10}v^2}{2}$ 0.5 pt.

so $v = 1.06 \times 10^7\,\mathrm{m \cdot s^{-1}}$. 0.5 pt.

The ^{10}Be ions pass through a region M with a homogeneous magnetic field.

ii. The ^{10}Be ions shall describe an exact quarter of a full circle with radius $R = 1\,\mathrm{m}$ in that region and fly straight into the tube located below region M. What is the required direction of the magnetic field? Indicate the direction with an arrow in the figure or describe! **1 pt.**

Considerung Lorentz' law $F = qv \times B$, the magnetic field has to point away from you into the figure. 1 pt.

iii. Calculate the required magnetic field strength. **2 pt.**

The Lorentz force need to be equal to the centripetal force,

$$qv \times B = m_{10}\frac{v^2}{R}$$

1 pt.

Therefore we have

$$v = \frac{qBR}{m_{10}}$$

0.5 pt.

The numerical value is $B = 1.13\,\mathrm{T}$. 0.5 pt.

After the tube the ^{10}Be ions enter the space between two charged metal plates E forming an exact quarter of a full circle with Radius $1\,\mathrm{m}$.

iv. What is the required direction of the electrical field lines between the plates to get the ^{10}Be ions to describe an exact quarter of a full circle? Indicate the field lines in the figure! **1 pt.**

The field lines need to be aligned radially pointing away from the center of the circle. 1 pt.

v. Calculate the required field strength. Can you give an approximate value for the charge that needs to be brought onto the plates to produce that field? Assume that the height of the plates is $10\,\mathrm{cm}$. **3 pt.**

We have $F = Eq$ 0.5 pt.

For the electrons to describe a circle the condition $Eq = m_{10}\frac{v^2}{R}$ needs to be fullfilled. Thus $E = 1.2 \times 10^7\,\mathrm{N \cdot C^{-1}}$ 0.5 pt.

For a plate capacitor we have $C = \varepsilon_0 \frac{A}{d}$, $Q = CU$ and $U = Ed$, which gives $Q = \varepsilon_0 \varepsilon A$ 1 pt.

With $A = R\frac{\pi}{2}h$, where h is the height of the capacitor ($A = 0.157\,\mathrm{m}^2$). 0.5 pt.

We get a numerical value of $Q = 1.62 \times 10^{-5}\,\mathrm{C}$ 0.5 pt.

The ^{10}Be ions which described an exact quarter of a full circle enter the detector D that counts each ion. During a measurement lasting one minute 2000 ^{10}Be ions have been counted

With a second detector I which can be put into the beam axis the ^{9}Be ions are counted. But because the abundance of these ions is much higher they cannot be counted as simgle ions. Instead, they are measured as a continous current (i.e each incoming ion produces an electron in the detector). For the above measurement, a current of $100\,\mathrm{nA}$ has been measured.

vi. Now calculate the ratio of the abundances of ^{10}Be over ^{9}Be ions in the rock sample. **1.5 pt.**

The ^{10}Be ions are coming out a rate of $c_{10} = \frac{2000}{60\,\mathrm{s}} = 33.3\,\mathrm{Hz}$. 0.5 pt.

On the other hand the rate for ^{9}Be ions is $c_9 = \frac{100\,\mathrm{nA}}{e} = 6.25 \times 10^{11}\,\mathrm{Hz}$ 0.5 pt.

We get a ratio $r = \frac{c_{10}}{c_9} = 5.33 \cdot 10^{-11}$. 0.5 pt.

vii. Let us assume that the ion source also produces ions with other masses and in other charge states (i.e two or more times negatively charged). Are these ions able to pass through the mass spectrometer and reach the detector D? Explain your answer! **1.5 pt.**

Other ions are able to pass through the spectrometer if they meet certain conditions. From the above calculations we have $\frac{B^2R^2}{2U} = \frac{m}{q}$ where the left side is a constant. Therefore ions with $\frac{m}{q} = \frac{m_{10}}{1e}$ can come trough the magnet. 1 pt.

For the charged plates we found $\frac{ER}{2U} = 1$, which means that all ions can go through the plates (if they passed the magnet) 0.5 pt.