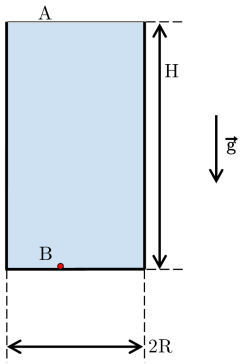
Challenge 6, Hydrodynamics

Submission date: 6. December

Hydrodynamics *(8 points)*

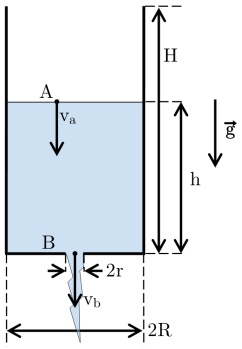
Part A. Perforated cylinder *(4 points)*

Be there a cylinder with height H and diameter $2R$, which is located at a place where standard pressure P_{atm} and gravity $\mathbf{g}$ apply. At the beginning, the cylinder is filled to the brim with an ideal, incompressible liquid of density ρ.



i. (1 pt.) Determine the **absolute** pressure P_b at the bottom of the cylinder (point B).

At time t_0, an hole with diameter $2r$ is pierced in the bottom of the cylinder. For simplicity, we assume that $r \ll R$.



ii. (1 pt.) Determine the velocity of the liquid surface in the cylinder (v_a) as a function of the velocity of the liquid passing through the hole (v_b) for a time $t > t_0$.

iii. (2 pt.) Find an expression for the velocity at the exit of the hole v_b as a function of the height h of fluid in the cylinder, taking into account the hypotheses given in the problem.

Part B. Hole in the swimming pool *(4 points)*

Blaise has designed and installed a new swimming pool in his garden. The pool is a perfect cylinder placed on the ground with the following dimensions: Diameter 1 m and height 1.5 m. Blaise fills the basin completely with water.

i. (2 pt.) Evangelista, Blaise's little brother, drills a hole in the wall of the pool at the height h above the ground, whereupon the water begins to flow out. How fast does the water flow out of the hole? Justify with a calculation.

ii. (1 pt.) What horizontal distance (from the hole) has a drop of water travelled when it touches the ground?

iii. (1 pt.) At what height above the ground should Evangelista drill a hole so that the droplet travels the furthest possible horizontal distance from the hole until it hits the ground?