Hydrodynamics

Warm-Up questions

Hydrodynamics

i. Viviane and Sebastian would like to build a submarine out of an old water tank.

a) **Assume the water tank has a volume of 2000 litres. What is the minimal weight of the submarine such that it can dive?**

b) **They would like to dive down to a depth of** 20m**. What is the water pressure at this depth?**

c) **To look out of the submarine, they build a round window with radius** $r = 20\text{cm}$**. What force is acting on the window in the depth of** 20m**?**

a) They float if their mass is smaller than the mass of the water contained in the volume of the submarine. Or equivalently if their mean density is smaller than the density of water. Since water has a density of $1000\text{kg} \cdot \text{m}^{-3}$, and since they want to dive and not to float, the minimal mass is 2000kg.

b) The pressure p at a distance h under the water surface is $p = \rho g h$ where $\rho = 1000\text{kg} \cdot \text{m}^{-3}$ is the density of water and $g = 9.81\text{m} \cdot \text{s}^{-1}$ the gravitational constant. Therefore the pressure when diving at a depth of 20m is $p = 1.96 \cdot 10^5\text{Pa} = 1.96\text{bar}$.

c) The force F is $F = pA$ where $A = \pi r^2$ is the area of the window.

ii. When it is raining, the water on a roof top is collected by a rain pipe and flows through a vertical downpipe to the ground. We assume a house with a 5m **long downpipe and we neglect any kind of friction.**

a) **How fast is the water flowing at the end of the downpipe?**

b) **At the lower end of the downpipe we place a water wheel. Assuming there are** 10 **litres of water per minute flowing through the pipe, what power can the water wheel produce (under ideal conditions)?**

a) Each drop with mass m floating from the roof to the ground picks up the potential energy $E_{\text{pot}} = mgh$ (where h is the height) and converts it into kinetic energy $E_{\text{kin}} = 1/2mv^2$. Since this happens simultaneously for all drops (cancelling the mass m), the water floats with a speed of

$$v = \sqrt{2gh} = 9.9\text{m} \cdot \text{s}^{-1}$$

b) A water stream of 10 litres per minute corresponds to a moved mass of $m = 10 \cdot \text{kg}$ per minute. The total energy of the water passing the water wheel in one minute is therefore

$$E_{\text{tot}} = \frac{1}{2}mv^2 = mgh = 490.5\text{J}.$$

Dividing by one minute (60s), we get a power of 8.18W.

iii. In this question we want to investigate the Bernoulli effect. For this we consider a pipe system with a narrowing, see figure 1. At the narrowest position, the diameter is half of the one at the beginning and the end. A little baby submarine is floating in the water, it has therefore always the same speed as the water. We assume the water floats at the widest positions with a speed v_0 and we neglect friction.

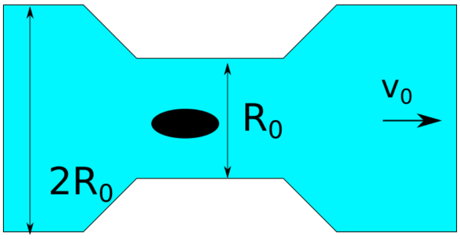

Figure 1:

a) **What is the speed at the narrowest position?**

b) **When floating from the widest to the narrowest position, the submarine gets accelerated, therefore the kinetic energy increases. How much is the increase assuming the submarine has a mass of m?**

c) **Where does the energy for this acceleration come from? Compare with the Bernoulli equation.**

a) The volume per time floating though the widest positions is the same as the volume per time passing the narrowest parts. This is the continuity equation which leads to $v_0\pi(2R_0)^2 = v_i\pi R_0^2$ where $v_i = 4v_0$ is the speed in the narrowest location.

b) The kinetic energy at the widest sections is $0.5mv_0^2$ and at the narrowest sections $0.5m(4v_0)^2$. Therefore the energy increases by a factor 16 or an absolute increase of $15/2mv_0^2$.

c) The energy comes from the water pressure being higher at the wide sections and lower at the smaller sections. At the passage from the widest to the narrowest section, the submarine gets pushed by the higher pressure in the widest part towards the lower pressure in the narrowest part. To compute the pressure, we use Bernoulli's equation $p_0 + 0.5\rho v_0^2 = p_i + 0.5\rho v_i^2$ where ρ is the water density and the index i denotes the pressure and speed in the narrowest area. Using $v_i = 4v_0$ and comparing the pressure difference $p_0 - p_i = 0.5\rho(v_i^2 - v_0^2) = 15/2\rho v_0^2$. Using that the density of the submarine is the same as of water (otherwise it would not float), and multiplying the pressure difference by the volume of the submarine $V = m/\rho$, we get $V(p_0 - p_i) = 15/2mv_0^2$. The energy therefore comes from the pressure difference. This is not surprising as the Bernoulli equation was derived from energy conservation.