Challenge 1, Mechanics: Solution

Along the rails *16 pt.*

The train G511 travels between Beijing and Wuhan. It takes a time T of 5 hours and 15 minutes to travel the distance D $1233\,\mathrm{km}$ of the journey at a cruise speed v_c of $300\,\mathrm{kmh^{-1}}$. There are 6 stops which respective time are 4, 5, 5, 6, 4 and 3 minutes (name them t_{si} with $i \in 1, 2, 3, 4, 5, 6$). Our goal is to estimate the acceleration a_T of the train with these data.

Please answer algebraically to all questions unless staten otherwise.

Part A. Before departure *2 pt.*

To start on a good basis let's clarify a few things.

i. What is the average speed at which the train moves during the travel? **1 pt.**

It is equal to the total distance divided by the total time minus the time spent at stations.

$$v_{average} = \frac{D}{T - \sum t_i}$$

1 pt.

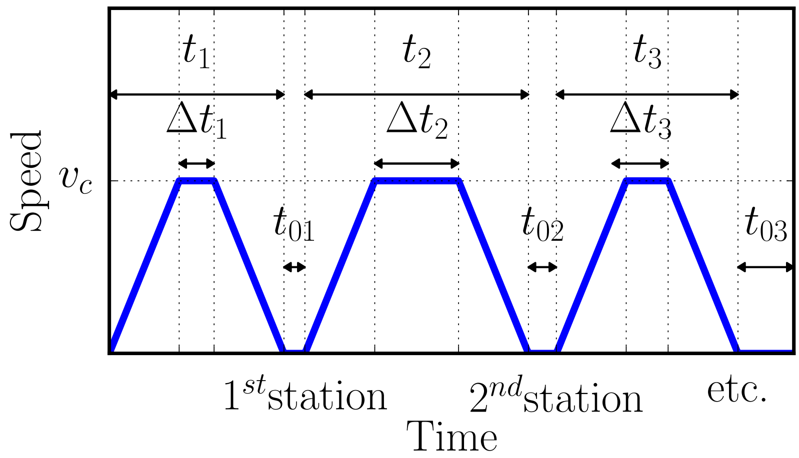
ii. Explain why the latter is different from the cruise speed. **1 pt.**

The cruise speed is the speed which is reached after acceleration, so the train usually doesn't travel at that speed, therefore the average speed is lower than the cruise speed. NB : these two questions are here to make sure the students understand what cruise speed is, and instead of giving it to them I introduce that with a question. 1 pt.

Part B. Travelling plan *6 pt.*

Assume that the train can reach its cruise speed for a finite time between each stations. ***Use algebraic values to illustrate your graphs. For example you can use t_i as departure time from each stations (with $t_0 = 0$ et $t_8 = T$).***

i. Sketch a graph of the speed of the train as a function of time for the first 3 stations after the departure station. **3 pt.**



Label of axes. 0.5 pt.

Name of intervals or at least labelling of intervals. 0.5 pt.

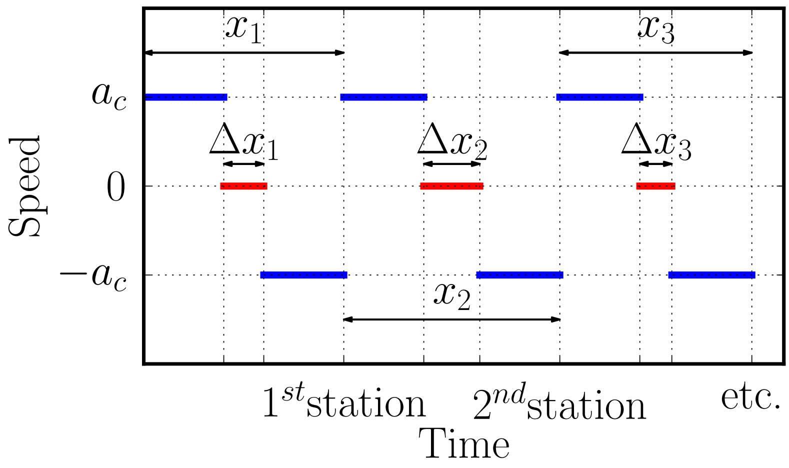
Non zero waiting time at the station during which the speed is 0. 0.5 pt.

Non zero time interval during which the speed is v_c between acceleration and deceleration. 1 pt.

Coherent curves for acceleration. 0.5 pt.

here t_i is the time of departure from station i, Δt_i is the time during which the train is at its cruise speed between stations $i-1$ and i. At station i the train spend a time t_{0i}. whenever the train is accelerating or decelerating the time elapsed will be called Δt

ii. Sketch a graph of the acceleration a_T of the train as a function of the distance for the 3 first stations. **3 pt.**



Label of axes. 0.5 pt.

Name of intervals or at least labelling of intervals. 0.5 pt.

Non zero waiting time at the station during which the speed is 0. 0.5 pt.

Non zero time interval during which the speed is v_c between acceleration and deceleration. 1 pt.

Coherent curves for acceleration. 0.5 pt.

here x_i is the distance between station i and the departure station, Δx_i is the distance covered by the train at its cruise speed between stations $i-1$ and i. Whenever the train is accelerating or decelerating the distance travelled will be called Δx

Part C. On the way *8 pt.*

Assume the acceleration is equal to the deceleration in this part. The hypothesis of part B. still holds.

i. Express the distance between two adjacent stations $x_{i+1} - x_i$ as a function of acceleration, cruise speed and of the distance over which the train travel at its cruise speed. ***(Find an appropriate name for this variable).*** **3 pt.**

The train covers the same distance, or spend the same time during acceleration and deceleration, this can be obtained by combining

$$x(t) = \frac{1}{2} a_T t^2$$

and

$$v(t) = at$$

1 pt.

at time Δt, the train reached its cruise speed, $v_c = v(\Delta t)$

$$\Delta x = \frac{v_c^2}{2a_T}$$

1 pt.

therefore the distance $x_{i+1} - x_i$ is given by

$$x_{i+1} - x_i = 2\Delta x + \Delta x_{i+1} = \frac{v_c^2}{a_T} + \Delta x_{i+1}$$

1 pt.

ii. Express the the time elapsed between the departure from two adjacent stations $t_{i+1} - t_i$ as a function of acceleration, cruise speed and of the distance over which the train travels at its cruise speed. **3 pt.**

The train covers the same distance, or spend the same time during acceleration and deceleration, this can be obtained simply with

$$v(t) = at$$

at time Δt, the train reached its cruise speed, $v_c = v(\Delta t)$

$$\Delta t = \frac{v_c}{a_T}$$

1 pt.

therefore the time $t_{i+1} - t_i$ is given by

$$t_{i+1} - t_i = 2\Delta t + \Delta t_{i+1} + t_{si+1} = \frac{2v_c}{a_T} + \Delta t_{i+1} + t_{si+1}$$

2 pt.

iii. Find the acceleration a_T of the train. *Write algebraic answer before the numerical application.* **2 pt.**

We are given the total time T and the total distance D of the travel, we can sum up the individual components computed in the previous part and equate them to the latter.

$$D = \sum_{i=0}^{N}(x_{i+1} - x_i) = \frac{Nv_c^2}{a_T} + \sum_{i=0}^{N-1} \Delta x_{i+1}$$

0.5 pt.

$$T = \sum_{i=0}^{N}(t_{i+1} - t_i) = \frac{2Nv_c}{a_T} + \sum_{i=0}^{N-1} \Delta t_{i+1} + \sum_{i=0}^{N-2} t_{si+1}$$

0.5 pt.

We can calculate the value of $Tv_c - D$ to get rid of the sums $\sum_{i=0}^{N-1} \Delta x_{i+1}$ and $\sum_{i=0}^{N-1} \Delta t_{i+1}$

$$Tv_c - D = \frac{Nv_c^2}{a_T} + v_c \sum_{i=0}^{N-2} t_{si+1}$$

0.5 pt.

as indeed the relation $\Delta t_{i+1} v_c = \Delta x_{i+1}$ is valid for every i. This yields the following result

$$a_T = \frac{Nv_c^2}{Tv_c - D - v_c \sum_{i=0}^{N-2} t_{si+1}}$$

0.25 pt.

Numerical application with $N = 7$

$$\frac{7(300\,\mathrm{kmh^{-1}})^2}{(5.25\,\mathrm{h})(300\,\mathrm{kmh^{-1}}) - 1233\,\mathrm{km} - (300\,\mathrm{km/h})(0.45\,\mathrm{h})} =$$

$$= 3043.48\,\mathrm{kmh^{-}2} = 0.235\,\mathrm{ms^{-2}} = a_T$$

0.25 pt.