Mechanics 1

Warm-Up questions

Kinematics (Chapter 2.2)

i. Alice throws a ball vertically upwards. She wants the ball to reach the height of the school wall, which a classmate tells her is $7\,\mathrm{m}$ tall. Alice throws the ball from $1\,\mathrm{m}$ above the ground. We neglect all friction.

a) **Alice throws the ball upwards with an initial velocity of $18\,\mathrm{km/h}$. Does the ball reach the top of the school wall?**

b) **Which minimal initial velocity does the ball need to have so that it reaches the top of the school wall?**

c) **In this case, what would the final velocity of the ball upon impact with the ground?**

a) The ball raises up to the height $h = -0.5gt^2 + v_0 t + x_0$ with $x_0 = 1\,\mathrm{m}$, $v_0 = 18\,\mathrm{km/h} = 5\,\mathrm{m/s}$, $g = 9.81\,\mathrm{m/s^2}$ and t the time taken by the ball to reach its maximal height.

To find t we have to remember that the speed of the ball at the top of its trajectory is zero. So $v_{top} = v_0 - gt = 0\,\mathrm{m/s}$ and $t = v_0/g$.

Putting this result into the first equation, we get $h = 2.27\,\mathrm{m} < 7\,\mathrm{m}$. So, the ball doesn't reach the top of the wall.

b) As before $t = v_0/g$ and $h = -0.5gt^2 + v_0 t + x_0$, this time v_0 is unknown and $h = 7\,\mathrm{m}$. Solving the system of equation, we get $v_0 = \sqrt{2g(h - x_0)} = 10.8\,\mathrm{m/s}$.

c) After it reaches the top of its trajectory, the ball will move downwards. We again have both equations $x_1 = -0.5gt^2 - v_{top}t + h$ and $v_1 = v_{top} + gt$ with $x_1 = 0\,\mathrm{m}$, $v_{top} = 0\,\mathrm{m/s}$ and $h = 7\,\mathrm{m}$.

With the first equation, we find $t = \sqrt{2h/g}$. Hence $v_1 = \sqrt{2gh} = 11.7\,\mathrm{m/s}$.

ii. While Denis is using his salad spinner, he wonders at what speed the leaves are rotating at the perimeter of the salad spinner. The salad spinner has a diameter of $30\,\mathrm{cm}$ and completes 9 rotations in $2\,\mathrm{s}$.

a) **At what speed do the leaves rotate at the perimeter of the salad spinner?**

b) **What is their acceleration?**

Upon opening the salad spinner, Denis discovers that there are more leaves at the perimeter of the spinner than before. However, Denis has learned in school that the acceleration vector is opposed to the position vector, and should therefore be pointing to the center.

c) **Explain why the leaves have moved to the perimeter of the salad spinner.**

a) At the boundary, the salad moves a distance $d = N \cdot 2\pi r$ in $t = 2\,\mathrm{s}$, with $r = 0.15\,\mathrm{m}$ being the radius and $N = 9$ the number of rotations during the time interval t. Thus, the speed is $v = d/t = 4.24\,\mathrm{m/s}$.

b) The speed is constant, but the velocity not since the salad doesn't always move in the same direction. The acceleration is perpendicular to the movement, towards the centre of the circular motion. Its magnitude is $a = v^2/r = 120\,\mathrm{m/s^2}$.

c) The acceleration's vector is due to the force applied by the boundary to the salad. This force does not let the salad escape the salad dryer. In the other hand, the salad placed in the middle of the dryer can continue its motion in a straight line, until it reaches the dryer's boundary.

Dynamics (Chapter 2.3)

iii. A wooden block is placed on a ramp. The coefficient of static friction is μ_s is 0.6 and the coefficient of kinetic friction is μ_k is 0.4. The block weighs $2\,\mathrm{kg}$.

a) What is the maximal angle of inclination of the ramp so that the block doesn't slide down the ramp?

b) Assume the ramp is at the maximal angle of inclination calculated above. We lightly push the block. Describe the velocity of the block over time.

a) Three forces act on the block: the gravitational force, the support N exercised by the ramp and the friction. Their projection on an axis parallel to the ramp gives $ma_{\parallel} = mg\sin(\theta) - \mu_s N$ and their projection on a perpendicular axis gives $ma_{\perp} = N - mg\cos(\theta)$ where θ is unknown.

Since the block is at rest both $a_{\parallel}$ and $a_{\perp}$ are zero and we obtain: $\mu_s = \tan(\theta)$. So, the maximal angle is $\theta = \arctan(\mu_s) = 31.0°$.

b) Since the bloc is now moving, the dynamical friction coefficient μ_d should be used. It is lower than μ_s, so the friction force will not be big enough to counteract the gravitational force and the block will accelerate down the ramp.

iv. Fred is driving $60\,\mathrm{km/h}$ along a country road. Suddenly, a deer crosses the road and Fred slows down. After $1.5\,\mathrm{s}$, he is now driving at $10\,\mathrm{km/h}$ and the deer disappears. We know that the combined mass of Fred and the car are $800\,\mathrm{kg}$. What is the average applied force during the deceleration?

We can use Newton's law: $\sum F = ma = \frac{dp}{dt}$ with $p = mv$ the momentum. Since we are interested in the average force, we must divide the variation of the momentum by the time interval: $\langle F \rangle = (mv_1 - mv_0)/\Delta t = 7410\,\mathrm{N}$ with v_0 and v_1 the initial respectively final velocities.

v. We assume that the moon moves around the Earth in a circular orbit.

a) What is the velocity of the moon?

b) What is the period of revolution of the moon around the Earth?

Some useful information: the distance between the Earth and moon is $3.84 \times 10^5\,\mathrm{km}$ **and the mass of the Earth is** $5.97 \times 10^{24}\,\mathrm{kg}$**.**

a) The only force felt by the Moon is the gravitational force. Thus $F = GmM/r^2 = ma$ with G the gravitational constant, m the mass of the Moon, M the mass of the Earth and r the distance between them. The acceleration of the circular uniform motion is $a = v^2/r$, with v the speed of the Moon.

Using both equations, we get $v = \sqrt{GM/r} = 1020\,\mathrm{m/s}$.

b) The revolution period is the time the Moon takes to make one revolution around the Earth. $T = d/v = 2\pi r/v = 2.37 \cdot 10^6\mathrm{s} = 27.4\,\mathrm{days}$.

Work and Energy (Chapter 2.4)

vi. Two balls are affixed to the ends of a long rod. The balls weigh $2\,\mathrm{kg}$ **and** $3\,\mathrm{kg}$ **and the length of the rod is** $1\,\mathrm{m}$**. We assume the mass of the rod is negligible. The rod rotates around its center of mass at a speed of 10 rotations per minute.**

a) What is the rotational energy of the system?

b) What is the moment of inertia of the system?

c) What is the angular momentum of the system?

d) How do these quantities change if the rod rotates around its geometric center?

a) We first must find the centre of mass of the system. We place the x-axis along the bar, with the origin on the lightest ball. Then, the position of the centre of mass $x_c = \frac{\sum_i m_i r_i}{\sum_i m_i} = \frac{m_2 l}{m_1 + m_2} = 0.6\,\mathrm{m}$ with $l = 1\,\mathrm{m}$ the length of the bar $m_1 = 2\,\mathrm{kg}$ and $m_2 = 3\,\mathrm{kg}$.

The rotation energy is due to the kinetics energy of the rotating body, so $E_{rot} = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2) = \frac{\omega^2}{2}\left(m_1 x_c^2 + m_2 (l - x_c)^2\right) = 0.658\,\mathrm{J}$ where $\omega = 2\pi 10\,\mathrm{min}^{-1} = 1.05\,\mathrm{s}^{-1}$.

b) The moment of inertia is $I = \sum_i r_i^2 m_i = x_c^2 m_1 + (l - x_c)^2 m_2 = 1.2\,\mathrm{m}^2\mathrm{kg}$.

c) The angular momentum is $L = I\omega = 1.26\,\mathrm{m}^2\mathrm{kg/s}$.

d) All the computations are the same as before, with $x_c = l/2 = 0.5\,\mathrm{m}$. Hence $E_{rot} = 0.685\,\mathrm{J}$, $I = 1.25\,\mathrm{m}^2\mathrm{kg}$ and $L = 1.31\,\mathrm{m}^2\mathrm{kg/s}$.