Thermodynamics

Warm-Up questions

Basic vocabulary (Chapter 4.1-4.3,4.7,4.8)

i. Given $55\,\mathrm{g}$ of oxygen gas, find the total number of oxygen molecules as well as the moles of oxygen molecules. What is the advantage of using the unit of moles?

Oxygen has a molar mass of $M = 32\,\mathrm{g} \cdot \mathrm{mol}^{-1}$. This gives the amount of moles for the oxygen gas $n = \frac{m}{M} = 1.7\,\mathrm{mol}$. With the Avogadro constant one can find the absolute number $N = nN_A = 1.04 \cdot 10^{24}$.

ii. A box contains a gas at $20\,°\mathrm{C}$. The box is heated until the gas's internal energy doubles. What is the temperature of the gas now?

The internal energy is proportional to the temperature. But we have to convert the temperature to Kelvin scale first. We get

$$T = 2(20 + 273.15)\mathrm{K} = 586\,\mathrm{K} = 313\,°\mathrm{C}$$

iii. Determine which of the following things can influence the internal energy of a gas:

a) kinetic translational energy of the gas molecules

b) heat energy of the gas molecules

c) potential energy due to the attractions between the gas molecules

d) rotational energy of the gas molecules

Which of the following contribute to the internal energy of an ideal gas?

In general translation, rotation and potential energy contribute to the internal energy. Altough transfering heat changes the internal energy it is not a part of the internal energy. In the ideal gas model the molecules are assumed to be point-like particles without molecular forces. Therefore only the translation energy contributes to the internal energy.

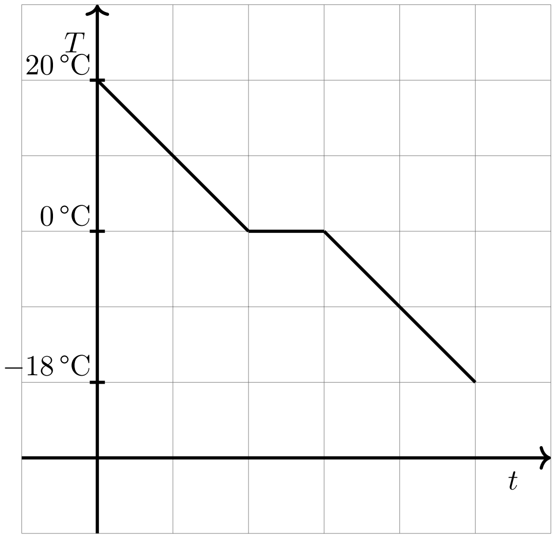
Heat capacity and phase changes (Chapter 4.4,4.13)

iv. You have water at room temperature ($20\,°\mathrm{C}$) and want to make a block of ice that will have a temperature of -$18\,°\mathrm{C}$.

a) Sketch the change in temperature as time passes.

b) You are using liquid nitrogen to cool the water. How much liquid nitrogen do you need to cool 100 grams of water into ice? Helpful quantities: specific heat capacity of water $c_w = 4.18\,\mathrm{J} \cdot \mathrm{g}^{-1} \cdot \mathrm{K}^{-1}$, specific heat capacity of ice $c_i = 2.05\,\mathrm{J} \cdot \mathrm{g}^{-1} \cdot \mathrm{K}^{-1}$, enthalpy of fusion / heat of fusion of water $L_w = 333.5\,\mathrm{J} \cdot \mathrm{g}^{-1}$, enthalpy of vaporization / heat of vaporization of nitrogen $L_n = 199\,\mathrm{J} \cdot \mathrm{g}^{-1}$.

a) The important thing is that the water stays at 0 °C during the phase transition from water to ice, so the temperatur curve will look something like



b) We first calculate how much heat per gram one needs to make the iceblock. We add all the contributions

$$c_{tot} = c_w \Delta T_w + L_w + c_i \Delta T_i = 454\,\mathrm{J} \cdot \mathrm{g}^{-1}$$

with $\Delta T_w = 20\,°\mathrm{C}$ and $\Delta T_i = 18\,°\mathrm{C}$. We can compare this to latent heat of nitrogen to get the mass of liquid nitrogen needed to make the iceblock.

$$m_n = \frac{c_{tot}}{L_n} m_w = 228\,\mathrm{g}$$

Ideal gas law (Chapter 4.5)

v. An air bubble is going up from the bottom of a water tank with height of $1\,\mathrm{m}$. The initial volume is $5\,\mathrm{cm}^3$. Considering that the water temperature is constant. What is its volume at the surface?

The pressure at the bottom is $p = \rho g h + p_{atm}$ at the surface $p = p_{atm}$. By the ideal gas law we get

$$V_{sur} = \frac{\rho g h + p_{atm}}{p_{atm}} V_0$$

We plug in the values and get

$$V_{sur} = \frac{1000\,\mathrm{kg} \cdot \mathrm{m}^{-3} \cdot 9.81\,\mathrm{m} \cdot \mathrm{s}^{-2} \cdot 1\,\mathrm{m} + 1 \times 10^5\,\mathrm{N} \cdot \mathrm{m}^{-2}}{1 \times 10^5\,\mathrm{N} \cdot \mathrm{m}^{-2}} \cdot 5\,\mathrm{cm}^3 = 5.5\,\mathrm{cm}^3$$

vi. We have an ideal gas with molar mass M.

a) **What is the relationship between the density and the temperature T? What is the relationship between the density and the pressure p?**

b) **As we know from experience, warm air rises. Does this also apply to ideal gases? If yes, why?**

a) The density ist $\rho = \frac{nM}{V}$ with n the numbers of particle in moles und V the volume of the gas. From the ideal gas law we get

$$\rho = \frac{nM}{V} = \frac{pM}{RT}$$

b) We seet that with higher temperature the gas becomes less dense. This means it experiences a buoyant force and ascends.

Processes and heat engines (Chapter 4.9-4.11)

vii. Which thermodynamic processes (isobaric, isochoric, isothermal, adiabatic) best describes the following phenomena:

a) the heating of a hot air balloon

b) Internal combustion of a diesel engine

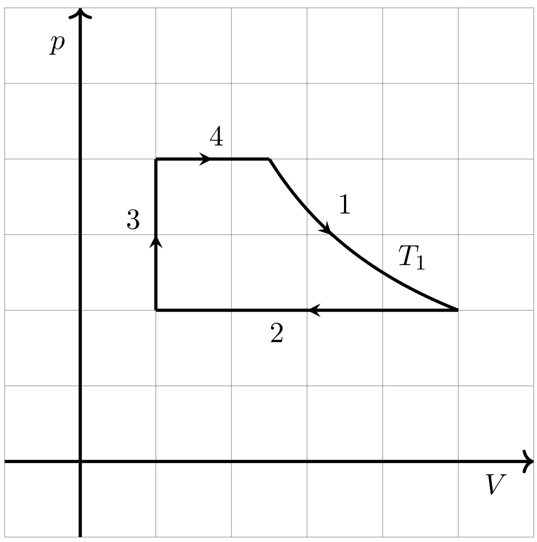
c) the inflation of a soccer ball

a) The ballon is open so pressure and volume stays constant. This means the process is isochoric and isobaric. Note, this doesn't mean that the temperature remains constant, because the system is open and the amount of particles can change as well.

b) The fuel burns so fast, that it cannot exchange heat with the environment. Therefore this is an adiabtic process.

c) The shape of the football remains more or less the same and also the tempature stays constant. Therefore the process is isochoric and isothermic. As in subquestion a) this system is open, which means the pressure can change even tough the volume and the temperature remain constant.

viii. The following thermodynamic cycle is described in a P-V diagram (Process 1 is isotherm bei Temperature T_1).



Qualitatively describe how the thermodynamic cycle would look in a T-V as well as a P-T diagram.

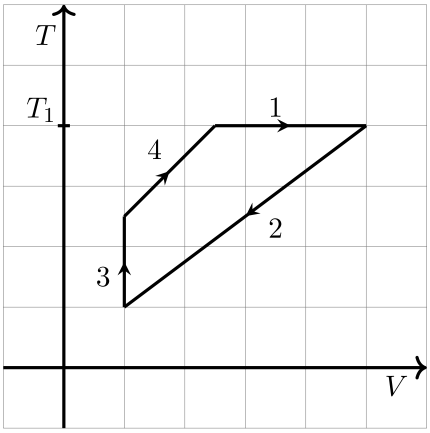
From the ideal gas law one can deduce how dependencies look like for the isobaric in the T-V diagram

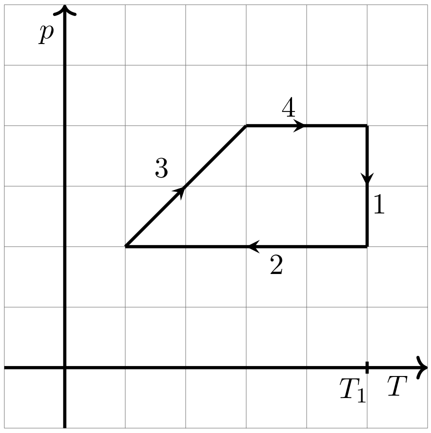
$$\left(\frac{p}{nR}\right) V = T$$

similarly we get for the isochoric process in the T-p diagram

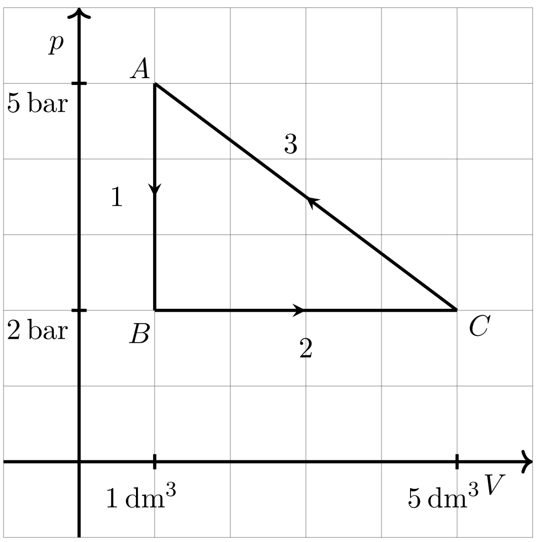
$$\left(\frac{V}{nR}\right) p = T$$

The processes where one of the variables p, V, T remains constant is perpendicular to the corresponding axis. With this one finds how the process looks like in the p-T and T-V diagram





ix. A heat engine that is filled with one mole of an ideal gas goes through the following thermodynamic cycle.



a) What type of processes are described by paths 1 and 2?

b) In which paths of the cycle is work done by the heat engine and in which paths is work done on the heat engine (from outside)? Calculate the amount (with the correct sign) for each path.

c) What is the net work done by the heat engine after one full cycle?

d) Calculate the external heat that is supplied to paths 1 and 3. What is the net heat flow after one full cycle?

a) process 1 is isochoric, process 2 isobaric

b) When the gas expands the machine gives mechanical work (process 2). When it contracts the machine consumes mechanical work (process 3). For process 1 the volume stays constant and therefore the machine neither gives nor consumes work. The quantitive number can be found by calculating the area below the curve of each subprocess

$$W_1 = 0, W_2 = -800\,\mathrm{J}, W_3 = 1400\,\mathrm{J}.$$

c) The netto work is just the sum of the work of all subprocesses

$$W = W_1 + W_2 + W_3 = 600\,\mathrm{J}.$$

d) By the first law of thermodynamics we have

$$\Delta U = W + Q$$

in each process. For an ideal gas the change in internal energy is related to change in temperature

$$\Delta U = \frac{3}{2} nR\Delta T,$$

which can also be related to the pressure and volume with the ideal gas law

$$T_i = \frac{p_i V_i}{nR}.$$

Combining all the formulas together we get

$$Q_i = \Delta U_i - W_i = \frac{3}{2}(p_{f,i}V_{f,i} - p_{i,i}V_{i,i}) - W_i,$$

where $p_{i,i}, V_{i,i}$ and $p_{f,i}, V_{f,i}$ are the initial respectively the final pressure and volume of each process. The numerical values are

$$Q_1 = -450\,\mathrm{J}, Q_2 = 2000\,\mathrm{J}, W_3 = -2150\,\mathrm{J}.$$

The netto heat is

$$Q = Q_1 + Q_2 + Q_3 = -600\,\mathrm{J}.$$

as expected, because we need to have $Q + W = \Delta U = 0$ for a cyclic process.