47th International Physics Olympiad
Switzerland Liechtenstein
Zurich, 11 – 17 July 2016

# exams

IPhO
2016
International
Physics Olympiad
Switzerland
Liechtenstein

## IPhO 2016 Exam Problems

The exams of the IPhO 2016 consisted of two experimental exams (to be solved in 5 hours) and 3 theoretical exams (to be solved in 5 hours). This document contains all exam problems of the IPhO 2016 and their general instructions. For brevity, the answer sheets and solutions are not included. The complete set of exams, instructions, answer sheets and solutions is available online at http://www.ipho2016.org/ipho2016/exam-and-program/.

**Experimental Exam Problems**

- General Instructions
- E-1: Electrical conductivity in two dimensions (10 points)
  by Matthias Hengsberger, Aram Kostanyan, Günther Palfinger (Idea Wafer)
- E-2: Jumping beads - A model for phase transitions and instabilities (10 points)
  by Christof Aegerter, Alex Kish

**Theoretical Exams Problems**

- General Instructions
- T-1: Two Problems in Mechanics (10 points)
  by Christoph Keller, Ben Kilminster, Andreas Schilling, Anton Alekseev, Johan Runeson
- T-2: Nonlinear Dynamics in Electric Circuits (10 points)
  by Anton Alekseev, Yves Barmaz, Pavel Rodin
- T-3: Large Hadron Collider (10 points)
  by Fritz Epple, Katharina Müller

# General instructions: Experimental Examination (20 points)

July 12, 2016

The experimental examination lasts for 5 hours and is worth a total of 20 points.

### Before the exam

- You must not open the envelopes containing the problems before the sound signal indicating the beginning of the competition.
- The beginning and end of the examination will be indicated by a sound signal. There will be announcements every hour indicating the elapsed time, as well as fifteen minutes before the end of the examination (before the final sound signal).

### During the exam

- Dedicated answer sheets are provided for writing your answers. Enter the observations into the appropriate tables, boxes or graphs in the corresponding answer sheet (marked A). For every problem, there are extra blank work sheets for carrying out detailed work (marked W). Be sure to always use the work sheets that belong to the problem you are currently working on (check the problem number in the header). If you have written something on any sheet which you do not want to be graded, cross it out. Only use to front side of every page.
- In your answers, try to be as concise as possible: use equations, logical operators and sketches to illustrate your thoughts whenever possible. Avoid the use of long sentences.
- Explicit error calculation is not required unless explicitly asked for. However, you are asked to give an appropriate number of significant digits when stating numbers. Also, you should decide on the appropriate number of data points or measurement repetitions unless specific instructions are given.
- You may often be able to solve later parts of a problem without having solved the previous ones.
- You are not allowed to leave your working place without permission. If you need any assistance (need to refill your drinking water bottle, broken calculator, need to visit a restroom, etc), please draw the attention of a team guide by putting one of the three flags into the holder attached to your cubicle ("Refill my water bottle, please", "I need to go to the toilet, please", or "I need help, please" in all other cases).

### At the end of the exam

- At the end of the examination you must stop writing immediately.
- For every problem, sort the corresponding sheets in the following order: cover sheet (C), questions (Q), answer sheets (A), work sheets (W).
- Put all the sheets belonging to one problem into the same envelope. Also put the general instructions (G) into the remaining separate envelope. Make sure your student code is visible in the viewing window of each envelope. Also hand in empty sheets. You are not allowed to take any sheets of paper out of the examination area.

- Put your writing equipment (2 ball point pens, 1 felt tip pen, 1 pencil, 1 pair of scissors, 1 ruler, 2 pairs of earplugs) as well as the provided calculator and your personal calculator (if applicable) back into the transparent zip bag.
- Wait at your table until your envelopes are collected. Once all envelopes are collected your guide will escort you out of the examination area. Take your writing equipment bag with you and hand it in at the exit. Also take your water bottle with you.

## Topics

| | | |
|---|---|---|
| **Experiment E-I**: | Electrical conductivity in two dimensions | 10 marks |
| **Experiment E-II**: | Jumping beads - A model for phase transitions and instabilities | 10 marks |

Experiments E-I and E-II share some of the same equipment. Among others, the same power supply and signal generator are used for both experiments, but with different settings.

Attention: when unpacking the box, do not lift the loudspeaker assembly by the plastic cylinder attached to the membrane.

## Material used in both experiments

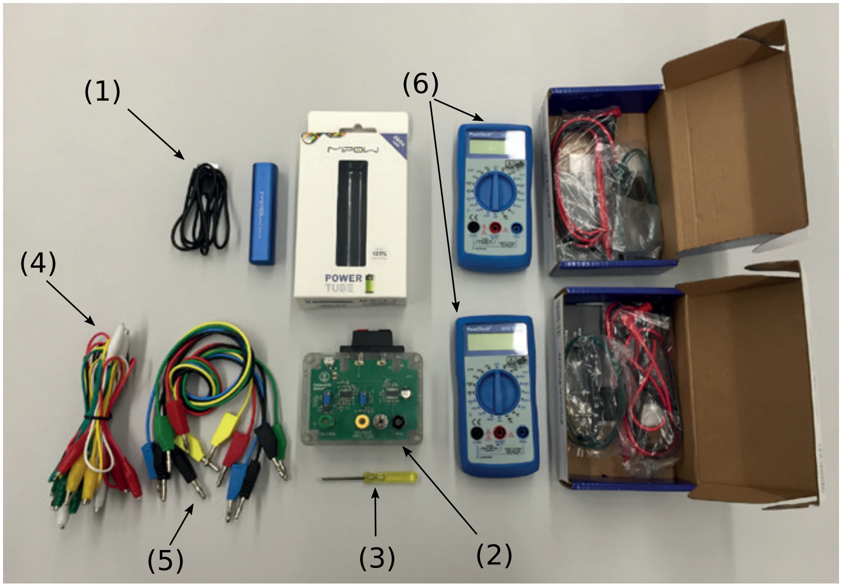



Figure 1: Common material for both experiments.

1. Battery pack with USB cable
2. Adjustable signal generator powered by the battery pack
3. Small screwdriver
4. Ten cables with crocodile clips
5. Six cables with $4\ \mathrm{mm}$ plugs
6. Two digital multimeters

You may also use any of the supplied stationary items to conduct the practical tasks.

### Signal generator

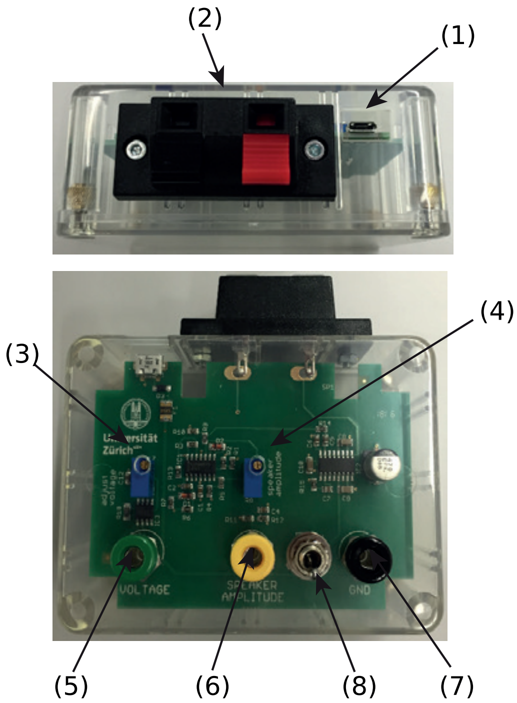



Figure 2.

1. USB connector for powering the signal generator
2. Loudspeaker terminals (only used in E-II)
3. Potentiometer for adjusting the constant voltage (only used in E-I)
4. Potentiometer for adjusting the speaker amplitude (only used in E-II)
5. DC voltage output socket (only used in E-I)
6. Monitor output socket for the loudspeaker drive amplitude (only used in E-II)
7. Common ground socket
8. Switch to turn the loudspeaker terminals and monitor output for the loudspeaker amplitude on / off

To power the signal generator, plug the battery pack using the USB cable to the USB connector of the signal generator (1).

Note that several turns of the potentiometer are required to go from one end of the range to the other. The potentiometers do not have mechanical stops at the end of their range.

### Digital multimeters

The digital multimeters can be used for current and voltage measurements. Always connect the two leads to the sockets labeled "$\mathrm{VmA\Omega}$" and "$\mathrm{GND}$" and choose current/voltage and the measurement range by means of the selector.

# Electrical conductivity in two dimensions (10 points)

Please read the general instructions in the separate envelope before you start this problem.

### Introduction

In the quest to develop next generation devices based on semi-conductor technology like computer chips or solar cells, researchers are looking for materials which exhibit outstanding transport properties, e.g. low electrical resistivity. Measurements of these properties are carried out using samples of finite size, contacts with finite contact resistance and in a special geometry. These effects have to be taken into account in order to extract the true material properties. Moreover, a thin film of the material may behave differently than bulk material.

In this task, we will investigate the measurement of electrical properties. We will use two different definitions:

- **Resistance $R$:** The resistance is the electrical property of a sample or device. It is the quantity which we actually measure on a specific sample with given dimensions.
- **Resistivity $\rho$:** The resistivity is the material property which determines the resistance. It depends on the material itself and on external parameters like the temperature, but it does not depend on the geometry of the sample.

In particular, we will measure the so-called *sheet resistivity*. This is the resistivity divided by the thickness of the very thin sheet.

We will explore the influence of the following parameters on the measurement of the electrical resistance of thin layers of material:

- the measurement circuitry,
- the measurement geometry,
- and the sample dimensions.

A sheet of conductive paper and a metal coated silicon wafer will serve as samples.

**List of materials**

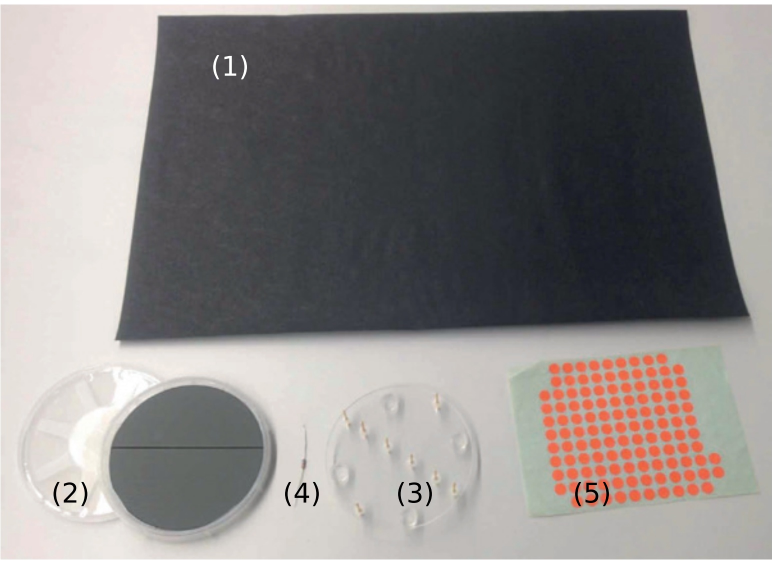



Figure 1: Additional equipment for this experiment.

1. Graphite coated conductive paper
2. A silicon wafer coated with a thin chromium film (stored in a wafer holder)
3. Plexiglas plate with 8 spring-loaded pins
4. An ohmic resistor
5. Color stickers

**Important precautions**

- The silicon wafer provided can easily be broken if dropped or bent. Do not touch or scratch the shiny metallic surface.

**Instructions**

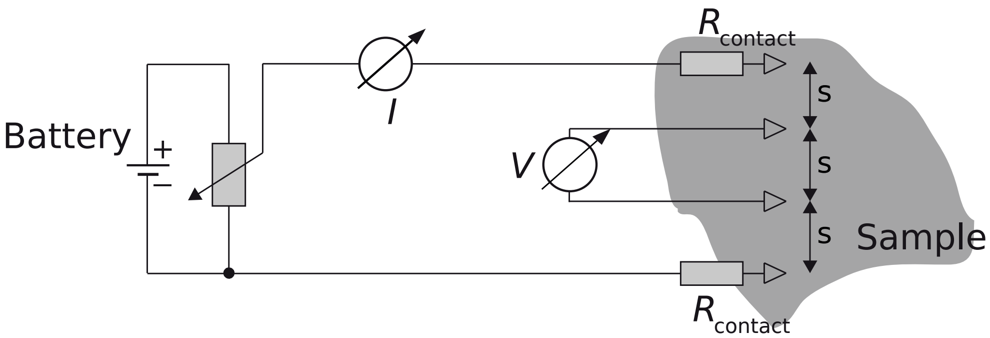
- In the experiment, the signal generator will be used as a DC voltage source. In this mode, the signal generator outputs a constant voltage between the *voltage* socket (5) and the *GND* socket (7). The numbers refer to the photograph shown in the general instructions.
- The voltage (range: $0$- $5\ \mathrm{V}$) can be adjusted on the left potentiometer labeled *adjust voltage* (3) using the screwdriver.
- When performing this experiment, make sure that the loudspeaker drive section of the signal generator is turned off using the toggle switch (8). This can be checked by measuring the voltage between the *speaker amplitude* monitor socket (6) and the *GND* socket (7). If the loudspeaker drive section is off, the voltage between these two terminals is zero.

## Part A. Four-point-probe (4PP) measurements (1.2 points)

In order to measure the resistivity of a sample precisely, the contacts used for the voltage measurement and the contacts used for current injection should be separated.

This technique is called four-point-probe technique (4PP). The four contacts are arranged into a symmetric geometry that is as simple as possible: The current $I$ flows into the sample through one of the outer contacts (called source), then on all possible paths through the sample and out of the sample through the other contact (drain). In between, the voltage $V$ is measured over a certain path length $s$ on the sample.

Everything becomes quite simple if we have a symmetric setup, i.e. the same distance $s$ between all contacts and the contacts in the center of the sample as shown in following sketch:




The curve $I$ *versus* $V$ represents the $I-V$-characteristics of the sample and allows the resistance of this sample segment to be determined. In the following we will only use the 4PP technique. To start, we will use the linear *equidistant* arrangement of four out of the eight probes (contacts) shown in the photograph.

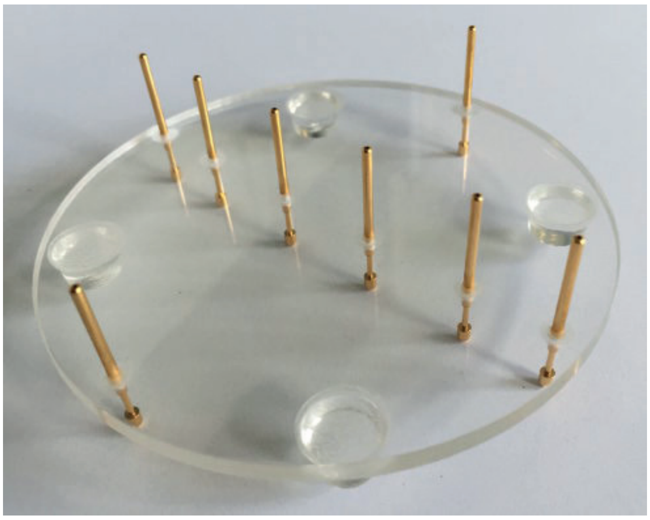


Figure 2: Acrylic glass plate for 4PP measurements, with the four rubber feet and the eight contacts or probes.

For the following measurement, use the whole sheet of conducting paper.

**Important hints for all following measurements:**

- The long side of the sheet of paper is the reference side. The four probes should be aligned parallel to this side.
- Be careful to use the coated side (black), not the brown back side of the paper! You may mark the correct orientation with color stickers.
- Check that there are no holes or cuts in the paper.
- For these measurements, place the contacts as close to the center of the sample as possible.
- Press the contacts with enough force to ensure good contact for each of them. The plastic feet should just touch the surface.

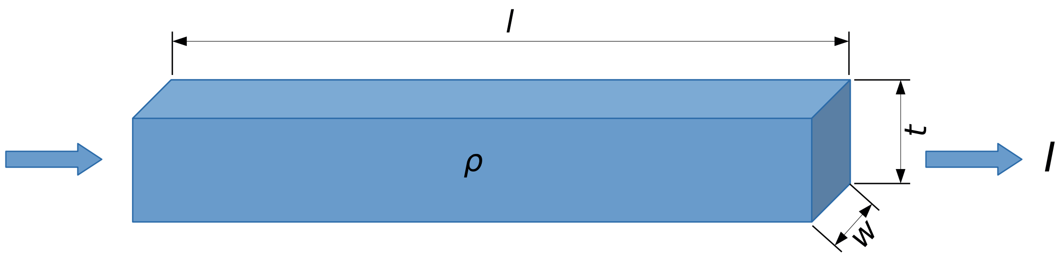
| | | |
|---|---|---|
| **A.1** | Four-point-probe (4PP) measurement: Measure the potential drop $V$ over a segment of length $s$ as function of current $I$ passing through this segment. Take in total at least 4 values, make a table and plot the voltage drop $V$ versus the current $I$ in **Graph A.1**. | 0.6pt |

| | | |
|---|---|---|
| **A.2** | Determine the effective electrical resistance $R = \frac{V}{I}$ that you obtained from **Graph A.1**. | 0.2pt |

| | | |
|---|---|---|
| **A.3** | Use **Graph A.1** to determine the uncertainty $\Delta R$ on the resistance $R$ for the 4PP measurement. | 0.4pt |

## Part B. Sheet resistivity (0.3 points)

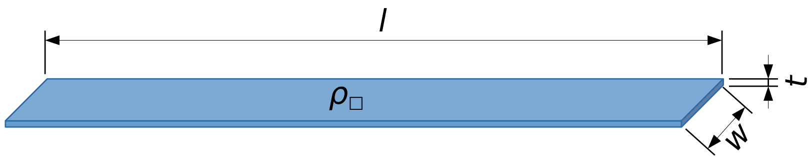
The resistivity $\rho$ represents a material property, by means of which the resistance of a 3D conductor of given dimensions and geometry is calculated. Here we consider a bar of dimensions length $l$, width $w$, and thickness $t$:




The electrical resistance $R$ of the upper, thick conductor is given by:

$$R = R_{\text{3D}} = \rho \cdot \frac{l}{w \cdot t} \tag{1}$$

On the same basis we may define the resistance of the 2D conductor of thickness $t \ll w$ and $t \ll l$




$$R = R_{\text{2D}} = \rho_{\square} \cdot \frac{l}{w}, \tag{2}$$

using the *sheet resistivity* $\rho_{\square} \equiv \rho/t$ ("rho box"). Its unit is given in Ohms: $[\rho_{\square}] = 1\ \Omega$.

**Important:** Eq. 2 is only valid for a homogeneous current density and constant potential in the cross-sectional plane of the conductor. In the case of point-like contacts on the surface this does not hold. Instead one can show that the sheet resistivity is related to the resistance in that case by

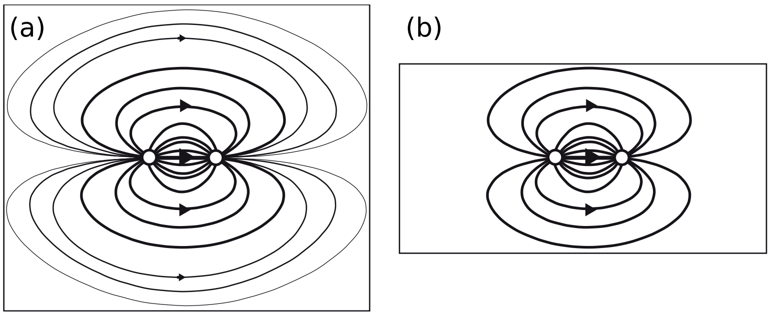
$$\rho_{\square} = \frac{\pi}{\ln(2)} \cdot R \tag{3}$$

for $l, w \gg t$.

| | | |
|---|---|---|
| **B.1** | Calculate the sheet resistivity $\rho_{\square}$ of the paper from the 4PP measurement in part A. We will call this particular value $\rho_{\infty}$ (and the measured resistance from part A $R_{\infty}$) because the sample dimensions of the whole sheet are much larger than the spacing of the contacts $s$: $l, w \gg s$. | 0.3pt |

## Part C. Measurements for different sample dimensions (3.2 points)

Up to now, the finite sample dimensions $w$ and $l$ were not taken into account. If the sample becomes smaller, it can carry less current if the voltage is kept constant: If we apply a voltage between the two point contacts (white circles), current will flow on all possible, non-crossing paths through the sample as visualized by the lines: the longer the line, the smaller the current as indicated by the line thickness. For a small sample (b) and the same applied voltage, the total current decreases because there are less possible pathways. Thus, the measured resistance will increase:




The (sheet) resistivity will not change as function of sample size. Thus, in order to convert the measured resistance into a resistivity using Eq. 3, we need to introduce a correction factor $f(w/s)$:

$$\rho_{\square} = \frac{\pi}{\ln(2)} \cdot \frac{R(w/s)}{f(w/s)}. \tag{4}$$

For a sample of length $l \gg s$ the factor $f$ only depends on the ratio $w/s$ and is larger than $1$: $f(w/s) \geq 1$. For the sake of simplicity we will focus on the dependence on the width $w$ and only ensure that the sample is long enough for our measurements. We assume that the value approaches the correct result $\rho_{\square}$ for large dimensions:

$$R(w/s) = R_{\infty} \cdot f(w/s) \quad \text{with} \quad f(w/s \to \infty) \to 1.0. \tag{5}$$

| | | |
|---|---|---|
| **C.1** | Using the 4PP-method, measure the resistance $R(w,s)$ for 4 values $w/s$ within the range 0.3 to 5.0 and record your results in **Table C.1**. Ensure that the sample length is larger than five times the probe spacing: $l > 5s$ and that the length $l$ of the samples is always taken along the same (long) side of the sheet of paper. For each value of $w/s$ measure the voltage for 4 different current values and calculate the average resistance $R(w/s)$ out of the 4 measurements. Enter your results in **Table C.1**. | 3.0pt |

| | | |
|---|---|---|
| **C.2** | Compute $f(w/s)$ for each of these measurements. | 0.2pt |

## Part D. Geometrical correction factor: scaling law (1.9 points)

You have seen in part C that the measured resistivity scales with the ratio of width to probe distance $w/s$. Starting from the data acquired in part C we choose the following generic function to describe the data in the range of the measurements:

$$\text{Generic fit function:} \quad f(w/s) = 1.0 + a \cdot \left(\frac{w}{s}\right)^b \tag{6}$$

Note that for very large $w/s$, $f(w/s)$ must be $1.0$.

| | | |
|---|---|---|
| **D.1** | In order to fit a model curve using Eq. 6 and the data $f(w/s)$, taken in part C, choose the most appropriate graph paper (linear **Graph D.1a**, semi-logarithmic **Graph D.1b**, or double-logarithmic **Graph D1.c**) to plot the data. | 1.0pt |

| | | |
|---|---|---|
| **D.2** | Deduce the parameters $a$ and $b$ from your fit. | 0.9pt |

## Part E. The silicon wafer and the van der Pauw-method (3.4 points)

In the semi-conductor industry, knowledge of the electrical (sheet) resistance of semi-conductors and thin metal layers is very important because it determines the properties of devices. In the following you will work with the silicon wafer. The semi-conducting wafer is coated with a very thin layer of chromium metal (on the shiny side).

Open the wafer container (rotate in the sense of the arrow RELEASE) and take the wafer out. Be careful not to drop or to break it nor to scratch or touch the shiny surface. For the measurements place it on the table with the shiny side point up towards you.

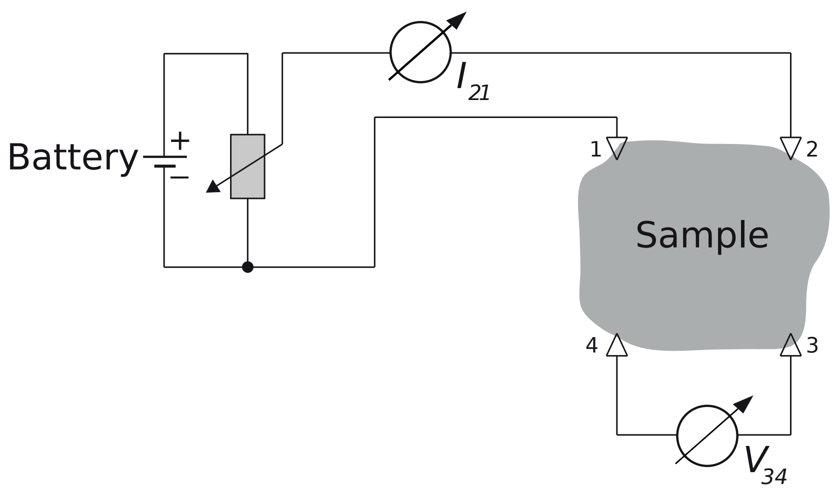
| | | |
|---|---|---|
| **E.1** | Use the same 4PP setup as previously to measure the voltage $V$ as function of current $I$.<br>Write down the reference number of your wafer in the Answer Sheet. You find this number on the plastic wafer holder. | 0.4pt |

| | | |
|---|---|---|
| **E.2** | Plot the data in **Graph E.2** and determine the resistance $R_{4\mathrm{PP}}$. | 0.4pt |

| | | |
|---|---|---|
| **E.3** | In order to determine the correction for a circular sample like the wafer, we will approximate the effective width $w$ of the sample by the diameter $D = 100\,\mathrm{mm}$ of the wafer. Under this assumption calculate the ratio $w/s$. Use the fit function in Eqn. 6 and your parameters $a$ and $b$ to determine the correction factor $f(w/s)$ for the wafer measurement. | 0.2pt |

| | | |
|---|---|---|
| **E.4** | Calculate the sheet resistivity $\rho_{\square}$ of the chromium layer using Eq. 4. | 0.1pt |

In order to measure the sheet resistivity precisely without need for geometrical corrections, Philips engineer L.J. van der Pauw developed a simple measurement scheme: The four probes are mounted at the circumference of a sample of arbitrary shape as shown in the figure (numbered 1 through 4). The current flows through two adjacent probes, e.g. probes 1 and 2, and the voltage is measured between probes 3 and 4. This yields a resistance value $R_{I,V} = R_{21,34}$.




For symmetry reasons $R_{21,34} = R_{34,21}$ and $R_{14,23} = R_{23,14}$. Van der Pauw showed that for an arbitrary but simply connected shape (no holes) of the sample and point-like contacts the following equation holds:

$$e^{-\pi R_{21,34}/\rho_\Box} + e^{-\pi R_{14,23}/\rho_\Box} \equiv 1. \tag{7}$$

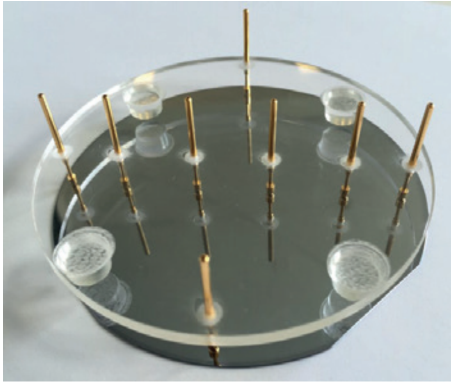


Figure 3: 4PP device on the metal coated silicon wafer. Note the cut on the right-hand-side of the circular wafer. This cut is called flat.

Connect the four spring contacts such that the measurement probes form a square. Connect two adjacent contacts to the current source with the amperemeter, and connect the two remaining spring contacts with the voltmeter. Rotate the square until one of its edges is parallel to the flat of the wafer.

| | | |
|---|---|---|
| **E.5** | Sketch the orientation of the current carrying contacts and the orientation of the flat of the wafer. Measure the voltage $V$ for at least in total 6 different values of current $I$, roughly equally spaced. Enter the results into **Table E.5**. | 0.6pt |

| | | |
|---|---|---|
| **E.6** | Repeat the procedure arranging the current carrying contacts perpendicular to those used in the first step. Enter the results into **Table E.6**. | 0.6pt |

| | | |
|---|---|---|
| **E.7** | Plot all the data together in a single graph **Graph E.7** using different colors and/or symbols. Determine the mean value $\langle R \rangle$ from the two curves. | 0.5pt |

| | | |
|---|---|---|
| **E.8** | Replacing all resistances $R_{kl,mn}$ by $\langle R \rangle$, solve Eqn. 7 for $\rho_\Box$ and calculate the sheet resistivity $\rho_\Box$ of the chromium layer. | 0.4pt |

| | | |
|---|---|---|
| **E.9** | Compare the result of the measurement taken with the linear arrangement (**E.4**) and the result of the van der Pauw method (**E.8**). Give the difference of the two measurements as relative error in percent. | 0.1pt |

| | | |
|---|---|---|
| **E.10** | The chromium (Cr) layers have a nominal thickness of $8\ \mathrm{nm}$. Use this value and the final results of the van der Pauw method to calculate the resistivity of Cr using Eqns. 1 and 2. | 0.1pt |

# Jumping beads - A model for phase transitions and instabilities (10 points)

Please read the general instructions in the separate envelope before you start this problem.

### Introduction

Phase transitions are well known from every day life, e.g. water takes different states like solid, liquid and gaseous. These different states are separated by phase transitions, where the collective behaviour of the molecules in the material changes. Such a phase transition is always associated with a transition temperature, where the state changes, i.e. the freezing and boiling temperatures of water in the above examples.

Phase transitions are however even more wide-spread and also occur in other systems, such as magnets or superconductors, where below a transition temperature the macroscopic state changes from a paramagnet to a ferromagnet and a normal conductor to a superconductor, respectively.

All of these transitions can be described in a common framework when introducing a so-called order-parameter. For instance, in magnetism the order parameter is associated with the alignment of the magnetic moments of the atoms with a macroscopic magnetisation.

In the so-called continuous phase transitions, the order parameter will always be zero above the critical temperature and then grow continuously below it, as shown in the schematic for a magnet in figure 1 below. The transition temperature of a continuous phase transition is called the critical temperature. The figure also contains a schematic representation of the microscopic order or disorder in the case of a magnet, where the individual magnetic moments align in the ferromagnetic state to give rise to a macroscopic magnetization, whereas they are randomly oriented in the paramagnetic phase yielding a macroscopic magnetization of zero.

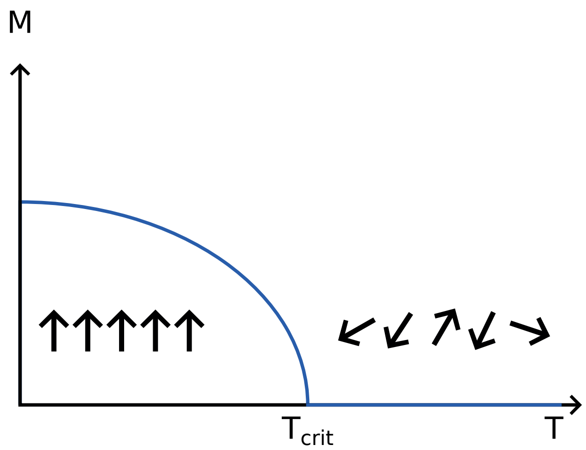



Figure 1: Schematic representation of the temperature dependence of an order parameter $M$ at a phase transition. Below the critical temperature $T_{\text{crit}}$, the order parameter grows and is non-zero, whereas it is equal to zero at temperatures above $T_{\text{crit}}$.

For continuous phase transitions, one generally finds that the order parameter close to a transition follows a power-law, e.g. in magnetism the magnetization $M$ below the critical temperature, $T_{\text{crit}}$, is given by:

$$M \begin{cases} \sim (T_{\text{crit}} - T)^b, & T < T_{\text{crit}} \\ = 0, & T > T_{\text{crit}} \end{cases} \quad (1)$$

where $T$ is temperature. What is even more stunning is that this behaviour is universal: the exponent of this power-law is the same for many different kinds of phase transition.

### Task

We will study a simple example where some of the features of continuous phase transitions can be investigated, such as how an instability leads to the collective behaviour of the particles and thus to the phase transition as well as how the macroscopic change depends on an excitation of the particles.

In common phase transitions this excitation is usually driven by temperature. In our example, the excitation consists of the kinetic energy of the particles accelerated by the loudspeaker. The macroscopic change corresponding to the phase transition that we study here consists of the sorting of beads into one half of a cylinder, which is separated by a small wall.

Increasing the amplitude from where particles have sorted into one half of the cylinder, you will find that eventually the particles distribute equally between the two halves. This corresponds to having heated past the critical temperature.

Your objective is to determine the critical exponent for the model phase transition studied here.

### List of material

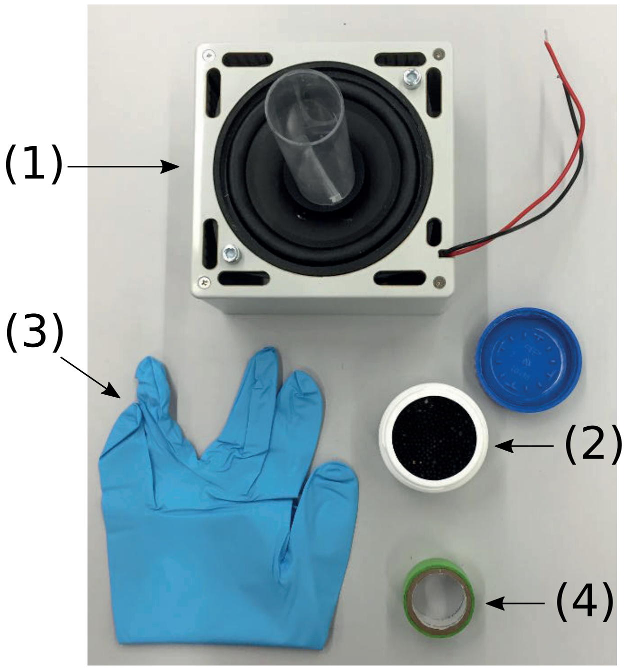



Figure 2: Additional equipment for this experiment.

1. Loudspeaker assembly with plastic cylinder mounted on top
2. About 100 poppy seeds (in a plastic container)
3. A glove
4. Sticky tape

**Important precautions**

- Do not apply an excessive lateral force to the plastic cylinder mounted on the loudspeaker. Note that no replacements will be provided in case of torn loudspeaker membranes or torn off plastic cylinder.
- Turn off the loudspeaker assembly whenever not in use, in order to avoid unnecessary drain of the battery.
- In this experiment, a $4\ \mathrm{Hz}$ saw-tooth signal is output on the loudspeaker terminals located at the side of the signal generator.
- The amplitude of the saw-tooth signal can be adjusted using the right potentiometer labeled *speaker amplitude* (4). A DC voltage proportional to the signal amplitude is output on the *speaker amplitude* monitor socket (6) (with respect to the *GND* socket (7)). The numbers refer to the photograph (Figure 2) shown in the general instructions.
- The speaker membrane is delicate. Make sure that you do not apply unnecessary pressure on it by any means either vertically or laterally.

## Part A. Critical excitation amplitude (3.3 points)

Before you start the actual tasks of this problem, wire up the loudspeaker to the terminals on the side of the signal generator (make sure you use the correct polarity). Put some (e.g. $50$) poppy seeds into the cylinder mounted on the loudspeaker and use a piece cut from the glove provided to close the cylinder at the top in order to keep the poppy seeds in the cylinder. Switch on the excitation using the toggle switch and adjust the amplitude by turning the right potentiometer labeled *speaker amplitude* (4) by means of the screwdriver provided. Observe the sorting of the beads by testing different amplitudes.

The first task is to determine the critical excitation amplitude of this transition. In order to do this, you have to determine the number of beads $N_1$ and $N_2$ in the two compartments (choosing the compartment labels such that $N_1 \leq N_2$) as a function of the displayed amplitude $A_D$, which is the voltage measured at the *speaker amplitude* socket (6). This voltage is proportional to the amplitude of the saw-tooth waveform driving the loudspeaker. Make at least 5 measurements per voltage.

Hint:

- In order to always have a motion in the particles you study, only investigate amplitudes corresponding to *speaker amplitude* voltages exceeding $0.7\ \mathrm{V}$. Start with watching the behaviour of the system just by varying the voltage slowly without any counting of he beads. It can be that some of the beads stick to the ground due to electrostatic reasons. Don't count these beads.

| | | |
|---|---|---|
| **A.1** | Record your measurements of the number of particles $N_1$ and $N_2$ in each half of the container for various amplitudes $A_D$ in **Table A.1**. | 1.2pt |

| | | |
|---|---|---|
| **A.2** | Calculate the standard deviation of your measurements of $N_1$ and $N_2$ and list your results in **Table A.1**. Plot $N_1$ and $N_2$ as a function of the displayed amplitude $A_D$ in **Graph A.2**, including their uncertainties. | 1.1pt |

| | | |
|---|---|---|
| **A.3** | Based on your graph, determine the critical displayed amplitude $A_{D,\mathrm{crit}}$ at which $N_1 = N_2$, after waiting until a stationary state is reached. | 1pt |

## Part B. Calibration (3.2 points)

The displayed amplitude $A_D$, corresponds to a voltage applied to the loudspeaker. However, the physically interesting quantity is the maximum displacement $A$ of the oscillation of the loudspeaker, since this relates to how strongly the beads are excited. Therefore, you need to calibrate the displayed amplitude. For this purpose, you can use any of the provided material and tools.

| | | |
|---|---|---|
| **B.1** | Sketch the setup you use to measure the excitation amplitude, i.e. the maximum travel distance $A$ (in $\mathrm{mm}$) of the loudspeaker in one period of oscillation. | 0.5pt |

| | | |
|---|---|---|
| **B.2** | Determine the amplitude $A$ in $\mathrm{mm}$ for a suitable number of points, i.e. record the amplitude $A$ as a function of displayed amplitude $A_D$ in **Table B.2** and indicate the uncertainties of your measurements. | 0.8pt |

| | | |
|---|---|---|
| **B.3** | Plot your data in **Graph B.3**, including the uncertainties. | 1.0pt |

| | | |
|---|---|---|
| **B.4** | Determine the parameters of the resulting curve, using an appropriate fit to determine the calibration function $A(A_D)$. | 0.8pt |

| | | |
|---|---|---|
| **B.5** | Determine the critical excitation amplitude $A_{\text{crit}}$ of the poppy seeds. | 0.1pt |

## Part C. Critical exponent (3.5 points)

In our system, the temperature corresponds to the input kinetic energy of the excitation. This energy is proportional to the speed squared of the loudspeaker, i.e. to $v^2 = A^2 f^2$, where $f$ is the frequency of the oscillation. We will now test this dependence and determine the exponent $b$ of the power-law governing the behavior of the order parameter (see Eq. 1).

| | | |
|---|---|---|
| **C.1** | The imbalance $\left\lvert \frac{N_1 - N_2}{N_1 + N_2} \right\rvert$ is a good candidate for an order parameter for our system in that it is zero above the critical amplitude and equal to $1$ at low excitation. Determine this order parameter as a function of the amplitude $A$. Record your results in the **Table C.1**. | 1.1pt |

| | | |
|---|---|---|
| **C.2** | Plot the imbalance $\left\lvert \frac{N_1 - N_2}{N_1 + N_2} \right\rvert$ as a function of $\left\lvert A_{\text{crit}}^2 - A^2 \right\rvert$, in **Graph C.2**, where both axes have logarithmic scales (double-logarithmic plot). You can use the **Table C.1** for your calculations. The points on the plot may seem not to obey a linear relation, but a linear regression should be made nevertheless, to match the critical exponent formula. | 1pt |

| | | |
|---|---|---|
| **C.3** | Determine the exponent $b$ and estimate the error. | 1.4pt |

# General instructions: Theoretical Examination (30 points)

July 14, 2016

The theoretical examination lasts for 5 hours and is worth a total of 30 points.

### Before the exam

- You must not open the envelopes containing the problems before the sound signal indicating the beginning of the examination.
- The beginning and end of the examination will be indicated by a sound signal. There will be announcements every hour indicating the elapsed time, as well as fifteen minutes before the end of the examination (before the final sound signal).

### During the exam

- Dedicated answer sheets are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding answer sheet (marked A). For every problem, there are extra blank work sheets for carrying out detailed work (marked W). Always use the work sheets that belong to the problem you are currently working on (check the problem number in the header). If you have written something on any sheet which you do not want to be graded, cross it out. Only use the front side of every page.
- In your answers, try to be as concise as possible: use equations, logical operators and sketches to illustrate your thoughts whenever possible. Avoid the use of long sentences.
- Please give an appropriate number of significant digits when stating numbers.
- You may often be able to solve later parts of a problem without having solved the previous ones.
- A list of physical constants is given on the next page.
- You are not allowed to leave your working place without permission. If you need any assistance (need to refill your drinking water bottle, broken calculator, need to visit a restroom, etc), please draw the attention of a team guide by putting one of the three flags into the holder attached to your cubicle ("Refill my water bottle, please", "I need to go to the toilet, please", or "I need help, please" in all other cases).

### At the end of the exam

- At the end of the examination you must stop writing immediately.
- For every problem, sort the corresponding sheets in the following order: cover sheet (C), questions (Q), answer sheets (A), work sheets (W).
- Put all the sheets belonging to one problem into the same envelope. Also put the general instructions (G) into the remaining separate envelope. Make sure your student code is visible in the viewing window of each envelope. Also hand in empty sheets. You are not allowed to take any sheets of paper out of the examination area.
- Leave the blue calculator provided by the organizers on the table.

- Take the writing equipment (2 ball point pens, 1 felt tip pen, 1 pencil, 1 pair of scissors, 1 ruler, 2 pairs of earplugs) as well your personal calculator (if applicable) with you. Also take your water bottle with you.
- Wait at your table until your envelopes are collected. Once all envelopes are collected your guide will escort you out of the examination area.

**General Data Sheet**

| | | | |
|---|---|---|---|
| Speed of light in vacuum | $c$ | $=$ | $299\ 792\ 458\ \mathrm{m \cdot s^{-1}}$ |
| Vacuum permeability (magnetic constant) | $\mu_0$ | $=$ | $4\pi \times 10^{-7}\ \mathrm{kg \cdot m \cdot A^{-2} \cdot s^{-2}}$ |
| Vacuum permittivity (electrical constant) | $\varepsilon_0$ | $=$ | $8.854\ 187\ 817 \times 10^{-12}\ \mathrm{A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}}$ |
| Elementary charge | $e$ | $=$ | $1.602\ 176\ 620\ 8(98) \times 10^{-19}\ \mathrm{A \cdot s}$ |
| Mass of the electron | $m_\mathrm{e}$ | $=$ | $9.109\ 383\ 56(11) \times 10^{-31}\ \mathrm{kg}$ |
| | | $=$ | $0.510\ 998\ 946\ 1(31)\ \frac{\mathrm{MeV}}{\mathrm{c}^2}$ |
| Mass of the proton | $m_\mathrm{p}$ | $=$ | $1.672\ 621\ 898(21) \times 10^{-27}\ \mathrm{kg}$ |
| | | $=$ | $938.272\ 081\ 3(58)\ \frac{\mathrm{MeV}}{\mathrm{c}^2}$ |
| Mass of the neutron | $m_\mathrm{n}$ | $=$ | $1.674\ 927\ 471(21) \times 10^{-27}\ \mathrm{kg}$ |
| | | $=$ | $939.565\ 413\ 3(58)\ \frac{\mathrm{MeV}}{\mathrm{c}^2}$ |
| Unified atomic mass unit | $u$ | $=$ | $1.660\ 539\ 040(20) \times 10^{-27}\ \mathrm{kg}$ |
| Rydberg constant | $R_\infty$ | $=$ | $10\ 973\ 731.568\ 508(65)\ \mathrm{m^{-1}}$ |
| Universal constant of gravitation | $G$ | $=$ | $6.674\ 08(31) \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ |
| Acceleration due to gravity (in Zurich) | $g$ | $=$ | $9.81\ \mathrm{m \cdot s^{-2}}$ |
| Planck's constant | $h$ | $=$ | $6.626\ 070\ 040\ (81) \times 10^{-34}\ \mathrm{kg \cdot m^2 \cdot s^{-1}}$ |
| Avogadro number | $N_\mathrm{A}$ | $=$ | $6.022\ 140\ 857\ (74) \times 10^{23}\ \mathrm{mol^{-1}}$ |
| Molar gas constant | $R$ | $=$ | $8.314\ 4598(48)\ \mathrm{kg \cdot m^2 \cdot s^{-2} \cdot mol^{-1} \cdot K^{-1}}$ |
| Molar mass constant | $M_\mathrm{u}$ | $=$ | $1 \times 10^{-3}\ \mathrm{kg \cdot mol^{-1}}$ |
| Boltzmann constant | $k_\mathrm{B}$ | $=$ | $1.380\ 648\ 52(79) \times 10^{-23}\ \mathrm{kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}}$ |
| Stefan-Boltzmann constant | $\sigma$ | $=$ | $5.670\ 367\ (13) \times 10^{-8}\ \mathrm{kg \cdot s^{-3} \cdot K^{-4}}$ |

# Two Problems in Mechanics (10 points)

Please read the general instructions in the separate envelope before you start this problem.

## Part A. The Hidden Disk (3.5 points)

We consider a solid wooden cylinder of radius $r_1$ and thickness $h_1$. Somewhere inside the wooden cylinder, the wood has been replaced by a metal disk of radius $r_2$ and thickness $h_2$. The metal disk is placed in such a way that its symmetry axis $B$ is parallel to the symmetry axis $S$ of the wooden cylinder, and is placed at the same distance from the top and bottom face of the wooden cylinder. We denote the distance between $S$ and $B$ by $d$. The density of wood is $\rho_1$, the density of the metal is $\rho_2 > \rho_1$. The total mass of the wooden cylinder and the metal disk inside is $M$.

In this task, we place the wooden cylinder on the ground so that it can freely roll to the left and right. See Fig. 1 for a side view and a view from the top of the setup.

The goal of this task is to determine the size and the position of the metal disk.

In what follows, when asked to express the result in terms of known quantities, you may always assume that the following are known:

$$r_1,\ h_1,\ \rho_1,\ \rho_2, M\ . \tag{1}$$

The goal is to determine $r_2, h_2$ and $d$, through indirect measurements.

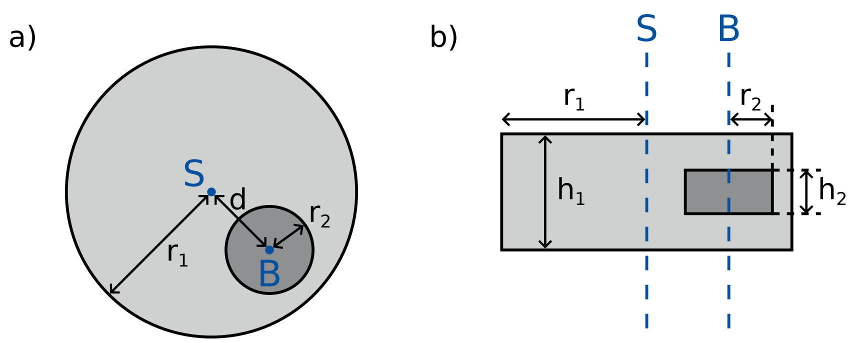



Figure 1: a) side view b) view from above

We denote $b$ as the distance between the centre of mass $C$ of the whole system and the symmetry axis $S$ of the wooden cylinder. In order to determine this distance, we design the following experiment: We place the wooden cylinder on a horizontal base in such a way that it is in a stable equilibrium. Let us now slowly incline the base by an angle $\Theta$ (see Fig. 2). As a result of the static friction, the wooden cylinder can roll freely without sliding. It will roll down the incline a little bit, but then come to rest in a stable equilibrium after rotating by an angle $\phi$ which we measure.

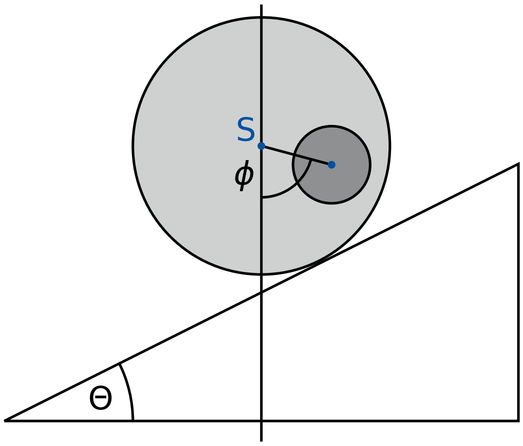


Figure 2: Cylinder on an inclined base.

| | | |
|---|---|---|
| **A.1** | Find an expression for $b$ as a function of the quantities (1), the angle $\phi$ and the tilting angle $\Theta$ of the base. | 0.8pt |

From now on, we can assume that the value of $b$ is known.

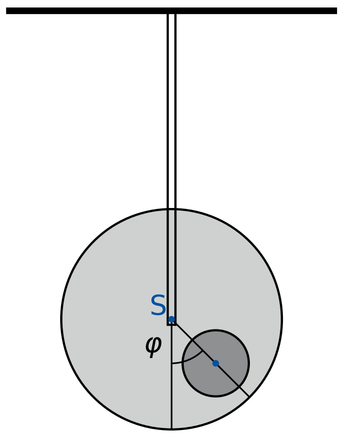


Figure 3: Suspended system.

Next we want to measure the moment of inertia $I_S$ of the system with respect to the symmetry axis $S$. To this end, we suspend the wooden cylinder at its symmetry axis from a rigid rod. We then turn it away from its equilibrium position by a small angle $\varphi$, and let it go. See figure 3 for the setup. We find that $\varphi$ describes a periodic motion with period $T$.

| | | |
|---|---|---|
| **A.2** | Find the equation of motion for $\varphi$. Express the moment of inertia $I_S$ of the system around its symmetry axis $S$ in terms of $T$, $b$ and the known quantities (1). You may assume that we are only disturbing the equilibrium position by a small amount so that $\varphi$ is always very small. | 0.5pt |

From the measurements in questions **A.1** and **A.2**, we now want to determine the geometry and the position of the metal disk inside the wooden cylinder.

| | | |
|---|---|---|
| **A.3** | Find an expression for the distance $d$ as a function of $b$ and the quantities (1). You may also include $r_2$ and $h_2$ as variables in your expression, as they will be calculated in subtask **A.5**. | 0.4pt |

| | | |
|---|---|---|
| **A.4** | Find an expression for the moment of inertia $I_S$ in terms of $b$ and the known quantities (1). You may also include $r_2$ and $h_2$ as variables in your expression, as they will be calculated in subtask **A.5**. | 0.7pt |

| | | |
|---|---|---|
| **A.5** | Using all the above results, write down an expression for $h_2$ and $r_2$ in terms of $b$, $T$ and the known quantities (1). You may express $h_2$ as a function of $r_2$. | 1.1pt |

## Part B. Rotating Space Station (6.5 points)

Alice is an astronaut living on a space station. The space station is a gigantic wheel of radius $R$ rotating around its axis, thereby providing artificial gravity for the astronauts. The astronauts live on the inner side of the rim of the wheel. The gravitational attraction of the space station and the curvature of the floor can be ignored.

| | | |
|---|---|---|
| **B.1** | At what angular frequency $\omega_{ss}$ does the space station rotate so that the astronauts experience the same gravity $g_E$ as on the Earth's surface? | 0.5pt |

Alice and her astronaut friend Bob have an argument. Bob does not believe that they are in fact living in a space station and claims that they are on Earth. Alice wants to prove to Bob that they are living on a rotating space station by using physics. To this end, she attaches a mass $m$ to a spring with spring constant $k$ and lets it oscillate. The mass oscillates only in the vertical direction, and cannot move in the horizontal direction.

| | | |
|---|---|---|
| **B.2** | Assuming that on Earth gravity is constant with acceleration $g_E$, what would be the angular oscillation frequency $\omega_E$ that a person on Earth would measure? | 0.2pt |

| | | |
|---|---|---|
| **B.3** | What angular oscillation frequency $\omega$ does Alice measure on the space station? | 0.6pt |

Alice is convinced that her experiment proves that they are on a rotating space station. Bob remains sceptical. He claims that when taking into account the change in gravity above the surface of the Earth, one finds a similar effect. In the following tasks we investigate whether Bob is right.

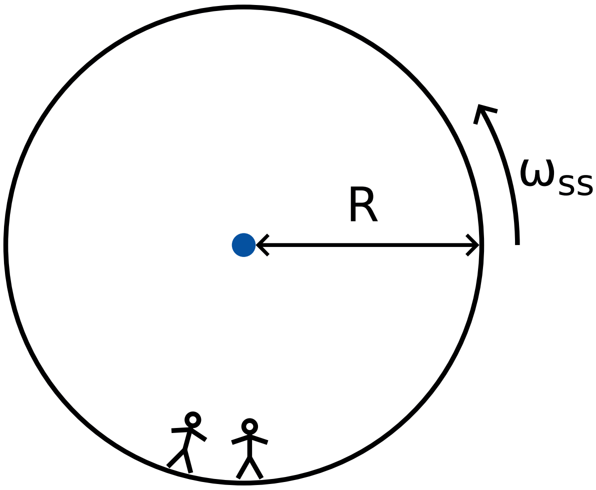



Figure 4: Space station

| | | |
|---|---|---|
| **B.4** | Derive an expression of the gravity $g_E(h)$ for small heights $h$ above the surface of the Earth and compute the oscillation frequency $\tilde{\omega}_E$ of the oscillating mass (linear approximation is enough). Denote the radius of the Earth by $R_E$. Neglect the rotation of Earth. | 0.8pt |

Indeed, for this space station, Alice does find that the spring pendulum oscillates with the frequency that Bob predicted.

| | | |
|---|---|---|
| **B.5** | For what radius $R$ of the space station does the oscillation frequency $\omega$ match the oscillation frequency $\tilde{\omega}_E$ on the Earth? Express your answer in terms of $R_E$. | 0.3pt |

Exasperated with Bob's stubbornness, Alice comes up with an experiment to prove her point. To this end she climbs on a tower of height $H$ over the floor of the space station and drops a mass. This experiment can be understood in the rotating reference frame as well as in an inertial reference frame.

In a uniformly rotating reference frame, the astronauts perceive a fictitious force $\vec{F}_C$ called the Coriolis force. The force $\vec{F}_C$ acting on an object of mass $m$ moving at velocity $\vec{v}$ in a rotating frame with constant angular frequency $\vec{\omega}_{ss}$ is given by

$$\vec{F}_C = 2m\vec{v} \times \vec{\omega}_{ss} \ . \qquad (2)$$

In terms of the scalar quantities you may use

$$F_C = 2mv\omega_{ss} \sin\phi \ , \qquad (3)$$

where $\phi$ is the angle between the velocity and the axis of rotation. The force is perpendicular to both the velocity $v$ and the axis of rotation. The sign of the force can be determined from the right-hand rule, but in what follows you may choose it freely.

| | | |
|---|---|---|
| **B.6** | Calculate the horizontal velocity $v_x$ and the horizontal displacement $d_x$ (relative to the base of the tower, in the direction perpendicular to the tower) of the mass at the moment it hits the floor. You may assume that the height $H$ of the tower is small, so that the acceleration as measured by the astronauts is constant during the fall. Also, you may assume that $d_x \ll H$. | 1.1pt |

To get a good result, Alice decides to conduct this experiment from a much taller tower than before. To her surprise, the mass hits the floor at the base of the tower, so that $d_x = 0$.

| | | |
|---|---|---|
| **B.7** | Find a lower bound for the height of the tower for which it can happen that $d_x = 0$. | 1.3pt |

Alice is willing to make one last attempt at convincing Bob. She wants to use her spring oscillator to show the effect of the Coriolis force. To this end she changes the original setup: She attaches her spring to a ring which can slide freely on a horizontal rod in the $x$ direction without any friction. The spring itself oscillates in the $y$ direction. The rod is parallel to the floor and perpendicular to the axis of rotation of the space station. The $xy$ plane is thus perpendicular to the axis of rotation, with the $y$ direction pointing straight towards the center of rotation of the station.

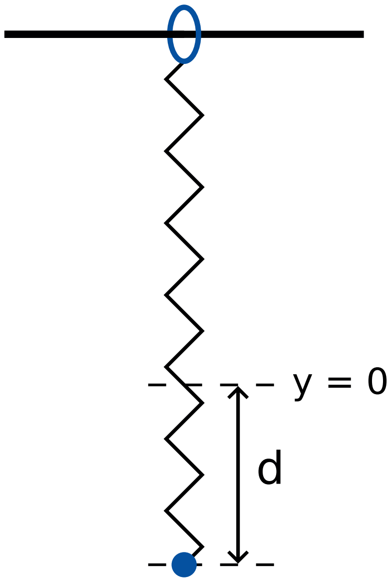



Figure 5: Setup.

| | | |
|---|---|---|
| **B.8** | Alice pulls the mass a distance $d$ downwards from the equilibrium point $x = 0$, $y = 0$, and then lets it go (see figure 5).<br>• Give an algebraic expression of $x(t)$ and $y(t)$. You may assume that $\omega_{ss} d$ is small, and neglect the Coriolis force for motion along the $y$-axis.<br>• Sketch the trajectory $(x(t), y(t))$, marking all important features such as amplitude. | 1.7pt |

Alice and Bob continue to argue.

# Nonlinear Dynamics in Electric Circuits (10 points)

Please read the general instructions in the separate envelope before you start this problem.

### Introduction

Bistable non-linear semiconducting elements (e.g. thyristors) are widely used in electronics as switches and generators of electromagnetic oscillations. The primary field of applications of thyristors is controlling alternating currents in power electronics, for instance rectification of AC current to DC at the megawatt scale. Bistable elements may also serve as model systems for self-organization phenomena in physics (this topic is covered in part B of the problem), biology (see part C) and other fields of modern nonlinear science.

### Goals

To study instabilities and nontrivial dynamics of circuits including elements with non-linear $I-V$ characteristics. To discover possible applications of such circuits in engineering and in modeling of biological systems.

## Part A. Stationary states and instabilities (3 points)

Fig. 1 shows the so-called **S-shaped** $I-V$ characteristics of a non-linear element $X$. In the voltage range between $U_{\mathrm{h}} = 4.00\ \mathrm{V}$ (the holding voltage) and $U_{\mathrm{th}} = 10.0\ \mathrm{V}$ (the threshold voltage) this $I-V$ characteristics is multivalued. For simplicity, the graph on Fig. 1 is chosen to be piece-wise linear (each branch is a segment of a straight line). In particular, the line in the upper branch touches the origin if it is extended. This approximation gives a good description of real thyristors.

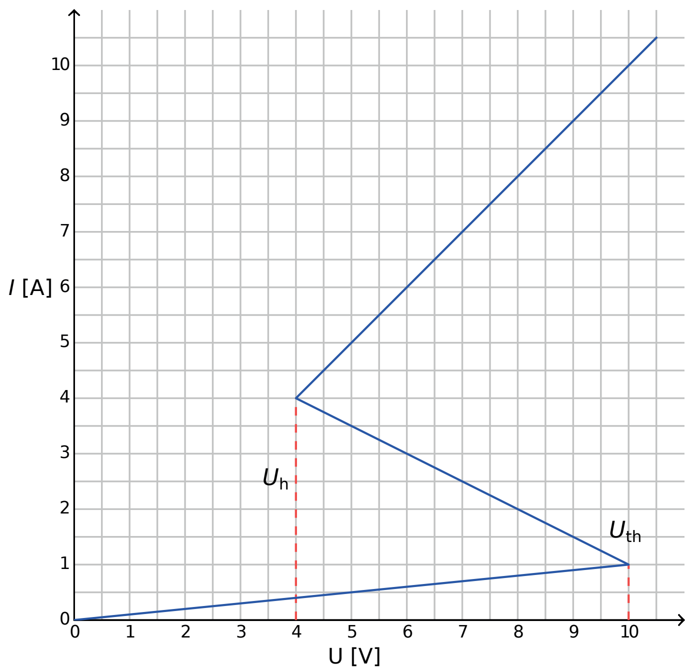



Figure 1: $I-V$ characteristics of the non-linear element $X$.

| | | |
|---|---|---|
| **A.1** | Using the graph, determine the resistance $R_{\text{on}}$ of the element $X$ on the upper branch of the $I-V$ characteristics, and $R_{\text{off}}$ on the lower branch, respectively. The middle branch is described by the equation $$I = I_0 - \frac{U}{R_{\text{int}}}. \quad (1)$$ Find the values of the parameters $I_0$ and $R_{\text{int}}$. | 0.4pt |

The element $X$ is connected in series (see Fig.2) with a resistor $R$, an inductor $L$ and an ideal voltage source $\mathcal{E}$. One says that the circuit is in a stationary state if the current is constant in time, $I(t) = \text{const}$.

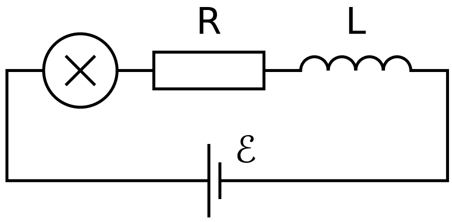



Figure 2: Circuit with element $X$, resistor $R$, inductor $L$ and voltage source $\mathcal{E}$.

| | | |
|---|---|---|
| **A.2** | What are the possible numbers of stationary states that the circuit of Fig. 2 may have for a fixed value of $\mathcal{E}$ and for $R = 3.00\ \Omega$? How does the answer change for $R = 1.00\ \Omega$? | 1pt |

| | | |
|---|---|---|
| **A.3** | Let $R = 3.00\ \Omega, L = 1.00\ \mu\text{H}$ and $\mathcal{E} = 15.0\ \text{V}$ in the circuit shown in Fig. 2. Determine the values of the current $I_{\text{stationary}}$ and the voltage $V_{\text{stationary}}$ on the non-linear element $X$ in the stationary state. | 0.6pt |

The circuit in Fig. 2 is in the stationary state with $I(t) = I_{\text{stationary}}$. This stationary state is said to be stable if after a small displacement (increase or decrease in the current), the current returns towards the stationary state. And if the system keeps moving away from the stationary state, it is said to be unstable.

| | | |
|---|---|---|
| **A.4** | Use numerical values of the question **A.3** and study the stability of the stationary state with $I(t) = I_{\text{stationary}}$. Is it stable or unstable? | 1pt |

## Part B. Bistable non-linear elements in physics: radio transmitter (5 points)

We now investigate a new circuit configuration (see Fig. 3). This time, the non-linear element $X$ is connected in parallel to a capacitor of capacitance $C = 1.00\ \mu\text{F}$. This block is then connected in series to a resistor of resistance $R = 3.00\ \Omega$ and an ideal constant voltage source of voltage $\mathcal{E} = 15.0\ \text{V}$. It turns out that this circuit undergoes oscillations with the non-linear element $X$ jumping from one branch of the $I-V$ characteristics to another over the course of one cycle.

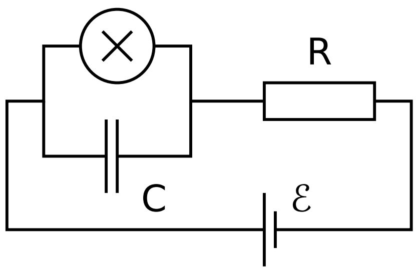



Figure 3: Circuit with element $X$, capacitor $C$, resistor $R$ and voltage source $\mathcal{E}$.

| | | |
|---|---|---|
| **B.1** | Draw the oscillation cycle on the $I - V$ graph, including its direction (clockwise or anticlockwise). Justify your answer with equations and sketches. | 1.8pt |

| | | |
|---|---|---|
| **B.2** | Find expressions for the times $t_1$ and $t_2$ that the system spends on each branch of the $I - V$ graph during the oscillation cycle. Determine their numerical values. Find the numerical value of the oscillation period $T$ assuming that the time needed for jumps between the branches of the $I - V$ graph is negligible. | 1.9pt |

| | | |
|---|---|---|
| **B.3** | Estimate the average power $P$ dissipated by the non-linear element over the course of one oscillation. An order of magnitude is sufficient. | 0.7pt |

The circuit in Fig. 3 is used to build a radio transmitter. For this purpose, the element $X$ is attached to one end of a linear antenna (a long straight wire) of length $s$. The other end of the wire is free. In the antenna, an electromagnetic standing wave is formed. The speed of electromagnetic waves along the antenna is the same as in vacuum. The transmitter is using the main harmonic of the system, which has period $T$ of question **B.2**.

| | | |
|---|---|---|
| **B.4** | What is the optimal value of $s$ assuming that it cannot exceed $1\ \mathrm{km}$? | 0.6pt |

## Part C. Bistable non-linear elements in biology: neuristor (2 points)

In this part of the problem, we consider an application of bistable non-linear elements to modeling of biological processes. A neuron in a human brain has the following property: when excited by an external signal, it makes one single oscillation and then returns to its initial state. This feature is called excitability. Due to this property, pulses can propagate in the network of coupled neurons constituting the nerve systems. A semiconductor chip designed to mimic excitability and pulse propagation is called a *neuristor* (from neuron and transistor).

We attempt to model a simple neuristor using a circuit that includes the non-linear element $X$ that we investigated previously. To this end, the voltage $\mathcal{E}$ in the circuit of Fig. 3 is decreased to the value $\mathcal{E}' = 12.0\ \mathrm{V}$. The oscillations stop, and the system reaches its stationary state. Then, the voltage is rapidly increased back to the value $\mathcal{E} = 15.0\ \mathrm{V}$, and after a period of time $\tau$ (with $\tau < T$) is set again to the value $\mathcal{E}'$ (see Fig. 4). It turns out that there is a certain critical value $\tau_{\mathrm{crit.}}$, and the system shows qualitatively different behavior for $\tau < \tau_{\mathrm{crit}}$ and for $\tau > \tau_{\mathrm{crit}}$.

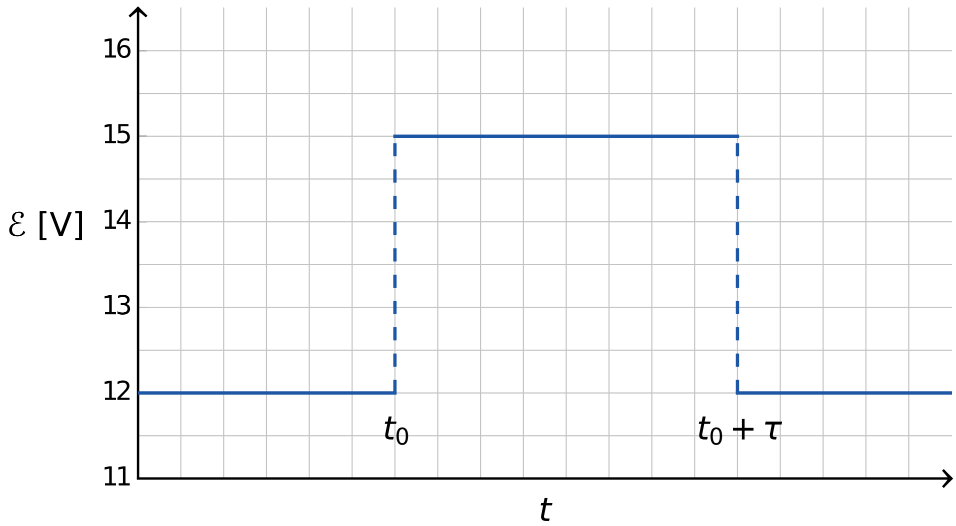



Figure 4: Voltage of the voltage source as a function of time.

| | | |
|---|---|---|
| **C.1** | Sketch the graphs of the time dependence of the current $I_X(t)$ on the non-linear element $X$ for $\tau < \tau_{\mathrm{crit}}$ and for $\tau > \tau_{\mathrm{crit}}$. | 1.2pt |

| | | |
|---|---|---|
| **C.2** | Find the expression and the numerical value of the critical time $\tau_{\mathrm{crit}}$ for which the scenario switches. | 0.6pt |

| | | |
|---|---|---|
| **C.3** | Is the circuit with $\tau = 1.00 \times 10^{-6}$ s a neuristor? | 0.2pt |

# Large Hadron Collider (10 points)

Please read the general instructions in the separate envelope before you start this problem.

In this task, the physics of the particle accelerator LHC (Large Hadron Collider) at CERN is discussed. CERN is the world's largest particle physics laboratory. Its main goal is to get insight into the fundamental laws of nature. Two beams of particles are accelerated to high energies, guided around the accelerator ring by a strong magnetic field and then made to collide with each other. The protons are not spread uniformly around the circumference of the accelerator, but they are clustered in so-called bunches. The resulting particles generated by collisions are observed with large detectors. Some parameters of the LHC can be found in table 1.

| **LHC ring** | |
|---|---|
| Circumference of ring | $26659\ \mathrm{m}$ |
| Number of bunches per proton beam | $2808$ |
| Number of protons per bunch | $1.15 \times 10^{11}$ |
| **Proton beams** | |
| Energy of protons | $7.00\ \mathrm{TeV}$ |
| Centre of mass energy | $14.0\ \mathrm{TeV}$ |

Table 1: Typical numerical values of relevant LHC parameters.

Particle physicists use convenient units for the energy, momentum and mass: The energy is measured in electron volts $[\mathrm{eV}]$. By definition, $1\ \mathrm{eV}$ is the amount of energy gained by a particle with elementary charge, e, moved through a potential difference of one volt ($1\ \mathrm{eV} = 1.602 \cdot 10^{-19}\ \mathrm{kg\,m^2s^{-2}}$).

The momentum is measured in units of $\mathrm{eV}/c$ and the mass in units of $\mathrm{eV}/c^2$, where $c$ is the speed of light in vacuum. Since $1\ \mathrm{eV}$ is a very small quantity of energy, particle physicists often use $\mathrm{MeV}$ ($1\ \mathrm{MeV} = 10^6\ \mathrm{eV}$), $\mathrm{GeV}$ ($1\ \mathrm{GeV} = 10^9\ \mathrm{eV}$) or $\mathrm{TeV}$ ($1\ \mathrm{TeV} = 10^{12}\ \mathrm{eV}$).

Part A deals with the acceleration of protons or electrons. Part B is concerned with the identification of particles produced in the collisions at CERN.

## Part A. LHC accelerator (6 points)

### Acceleration:

Assume that the protons have been accelerated by a voltage $V$ such that their velocity is very close to the speed of light and neglect any energy loss due to radiation or collisions with other particles.

| | | |
|---|---|---|
| **A.1** | Find the exact expression for the final velocity $v$ of the protons as a function of the accelerating voltage $V$, and physical constants. | 0.7pt |

A design for a future experiment at CERN plans to use the protons from the LHC and to collide them with electrons which have an energy of $60.0\ \mathrm{GeV}$.

| | | |
|---|---|---|
| **A.2** | For particles with high energy and low mass the relative deviation $\Delta = (c - v)/c$ of the final velocity $v$ from the speed of light is very small. Find a first order approximation for $\Delta$ and calculate $\Delta$ for electrons with an energy of $60.0\ \mathrm{GeV}$using the accelerating voltage $V$ and physical constants. | 0.8pt |

We now return to the protons in the LHC. Assume that the beam pipe has a circular shape.

| | | |
|---|---|---|
| **A.3** | Derive an expression for the uniform magnetic flux density $B$ necessary to keep the proton beam on a circular track. The expression should only contain the energy of the protons $E$, the circumference $L$, fundamental constants and numbers. You may use suitable approximations if their effect is smaller than precision given by the least number of significant digits.<br>Calculate the magnetic flux density $B$ for a proton energy of $E = 7.00\ \mathrm{TeV}$, neglecting interactions between the protons. | 1.0pt |

**Radiated Power:**

An accelerated charged particle radiates energy in the form of electromagnetic waves. The radiated power $P_{\mathrm{rad}}$ of a charged particle that circulates with a constant angular velocity depends only on its acceleration $a$, its charge $q$, the speed of light $c$ and the permittivity of free space $\varepsilon_0$.

| | | |
|---|---|---|
| **A.4** | Use dimensional analysis to find an expression for the radiated power $P_{\mathrm{rad}}$. | 1.0pt |

The real formula for the radiated power contains a factor $1/(6\pi)$; moreover, a full relativistic derivation gives an additional multiplicative factor $\gamma^4$, with $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$.

| | | |
|---|---|---|
| **A.5** | Calculate $P_{\mathrm{tot}}$, the total radiated power of the LHC, for a proton energy of $E = 7.00\ \mathrm{TeV}$ (Note table 1). You may use suitable approximations. | 1.0pt |

**Linear Acceleration:**

At CERN, protons at rest are accelerated by a linear accelerator of length $d = 30.0\ \mathrm{m}$ through a potential difference of $V = 500\ \mathrm{MV}$. Assume that the electrical field is homogeneous. A linear accelerator consists of two plates as sketched in Figure 1.

| | | |
|---|---|---|
| **A.6** | Determine the time $T$ that the protons take to pass through this field. | 1.5pt |

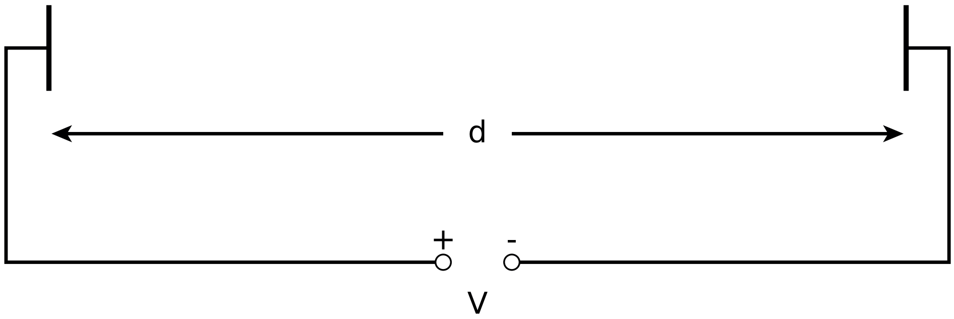



Figure 1: Sketch of an accelerator module.

## Part B. Particle Identification (4 points)

**Time of flight:**

It is important to identify the high energy particles that are generated in the collision in order to interpret the interaction process. A simple method is to measure the time ($t$) that a particle with known momentum needs to pass a length $l$ in a so-called Time-of-Flight (ToF) detector. Typical particles which are identified in the detector, together with their masses, are listed in table 2.

| **Particle** | **Mass** $[\mathrm{MeV/c^2}]$ |
|---|---|
| Deuteron | $1876$ |
| Proton | $938$ |
| charged Kaon | $494$ |
| charged Pion | $140$ |
| Electron | $0.511$ |

Table 2: Particles and their masses.

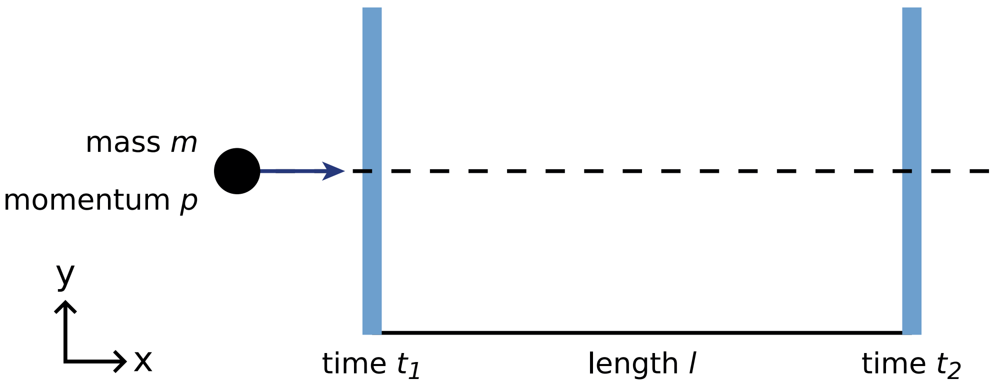
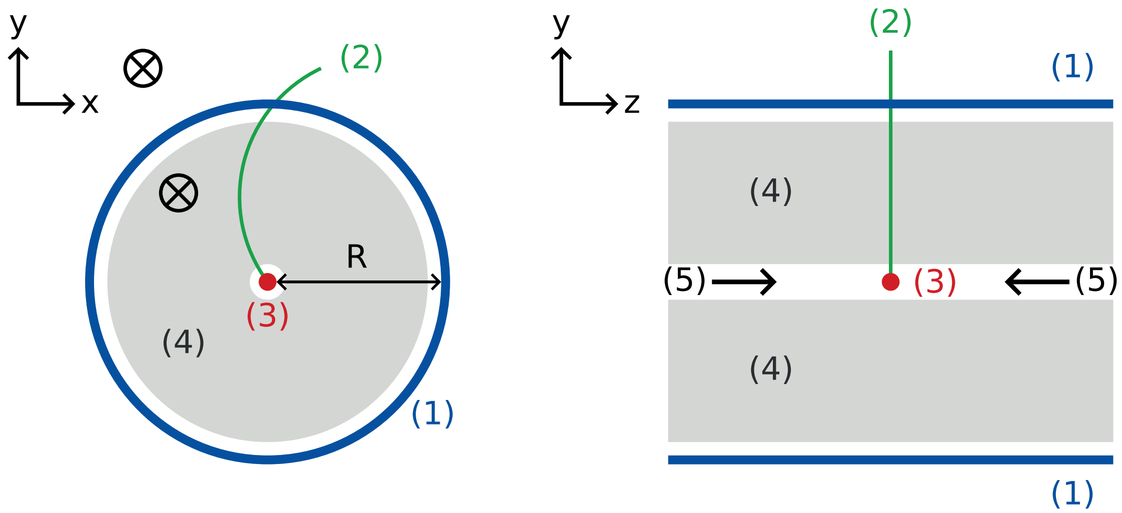



Figure 2: Schematic view of a time-of-flight detector.

| | | |
|---|---|---|
| **B.1** | Express the particle mass $m$ in terms of of the momentum $p$, the flight length $l$ and the flight time $t$, assuming that particles have elementary charge $e$ and travel with velocity close to $c$ on straight tracks in the ToF detector and that they travel perpendicular to the two detection planes (see figure 2). | 0.8pt |

| | | |
|---|---|---|
| **B.2** | Calculate the minimal length $l$ of a ToF detector that allows to safely distinguish a charged kaon from a charged pion, given both their momenta are measured to be $1.00\ \mathrm{GeV/c}$. For a good separation it is required that the difference in the time-of-flight is larger than three times the time resolution of the detector. The typical resolution of a ToF detector is $150\ \mathrm{ps}\ (1\ \mathrm{ps} = 10^{-12}\ \mathrm{s})$. | 0.7pt |

In the following, particles produced in a typical LHC detector are identified in a two stage detector consisting of a tracking detector and a ToF detector. Figure 3 shows the setup in the plane transverse and longitudinal to the proton beams. Both detectors are tubes surrounding the interaction region with the beam passing in the middle of the tubes. The tracking detector measures the trajectory of a charged particle which passes through a magnetic field whose direction is parallel to the proton beams. The radius $r$ of the trajectory allows one to determine the transverse momentum $\mathrm{p_T}$ of the particle. Since the collision time is known the ToF detector only needs one tube to measure the flight time (time between the collision and the detection in the ToF tube). This ToF tube is situated just outside the tracking chamber. For this task you may assume that all particles created by the collision travel perpendicular to the proton beams, which means that the created particles have no momentum along the direction of the proton beams.




transverse plane

cross section of the longitudinal view at the center of the tube along the beamline

(1) - ToF tube
(2) - track
(3) - collision point
(4) - tracking tube
(5) - proton beams
⊗ - magnetic field

Figure 3 : Experimental setup for particle identification with a tracking chamber and a ToF detector. Both detectors are tubes surrounding the collision point in the middle. Left : transverse view perpendicular to the beamline. Right : longitudinal view parallel to the beam line. The particle is travelling perpendicular to the beam line.

| | | |
|---|---|---|
| **B.3** | Express the particle mass in terms of the magnetic flux density $B$, the radius $R$ of the ToF tube, fundamental constants and the measured quantities: radius $r$ of the track and time-of-flight $t$. | 1.7pt |

We detected four particles and want to identify them. The magnetic flux density in the tracking detector was $B = 0.500\ \mathrm{T}$. The radius $R$ of the ToF tube was $3.70\ \mathrm{m}$. Here are the measurements ($1\ \mathrm{ns} = 10^{-9}\ \mathrm{s}$):

| *Particle* | *Radius of the trajectory* $r$ $[\mathrm{m}]$ | *Time of flight* $t$ $[\mathrm{ns}]$ |
|---|---|---|
| A | $5.10$ | $20$ |
| B | $2.94$ | $14$ |
| C | $6.06$ | $18$ |
| D | $2.31$ | $25$ |

| | | |
|---|---|---|
| **B.4** | Identify the four particles by calculating their mass. | 0.8pt |

**Imprint**

Editorial Team
Thomas Uehlinger
Marco Gerber

Exam Authors
Christof Aegerter
Anton Alekseev
Yves Barmaz
Fritz Epple
Matthias Hengsberger
Christoph Keller
Ben Kilminster
Alex Kish
Aram Kostanyan
Katharina Müller
Günther Palfinger
Pavel Rodin
Johan Runeson
Andreas Schilling

Graphic Design and Layout (Cover)
Stephanie Lippert
MELS, University of Zurich

Printing House
Mattenbach AG



International Physics Olympiad 2016
www.ipho2016.org
info@ipho2016.org

## Organizers

University of Zurich[UZH]

OFFICE OF EDUCATION
PRINCIPALITY OF LIECHTENSTEIN

Verband Schweizer Wissenschafts-Olympiaden
Association des Olympiades Scientifiques Suisses
Associazione delle Olimpiadi Scientifiche Svizzere
Association of Swiss Scientific Olympiads

SWISS PHYSICS OLYMPIAD

## Main Partner

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Swiss Confederation

Federal Department of Economic Affairs,
Education and Research EAER
**State Secretariat for Education,**
**Research and Innovation SERI**

## Promoting Partner

Metrohm Stiftung

## Partners

Baugarten Stiftung

CERN

the cogito foundation

ERNST GÖHNER STIFTUNG

## Partners

Roche

NOVARTIS

PAUL SCHERRER INSTITUT
PSI

FNSNF
Swiss National Science Foundation

Stadt Zürich

SwissMAP
The Mathematics of Physics
National Centre of Competence in Research

UZH FOUNDATION
The Foundation of the University of Zurich

### Supporters: Institutions and Public Sector

Swisslos – Kanton Aargau
University of Berne

### Supporters: Companies

Accor Hotels SA
business campaigning GmbH
Bruker BioSpin AG
EUROBUS welti furrer AG
Hoval AG
IBM Research GmbH – IBM Research – Zurich
Interface Marketing AG
Lake Lucerne Navigation Company
Mount Rigi Railways SA
Sensirion AG
SIGG Switzerland AG
ZFV Group

### Supporters: Foundations

Gedächtnisstiftung Peter Kaiser
Georg H. Endress Foundation
Hilti Foundation
Karl Mayer Foundation
Lotex Foundation
Maiores Foundation
Polianthes Foundation
Prof. Otto Beisheim Foundation
RHW Foundation
SwissLife Foundation “Perspectives”